X-ray Timing in Astrophysics

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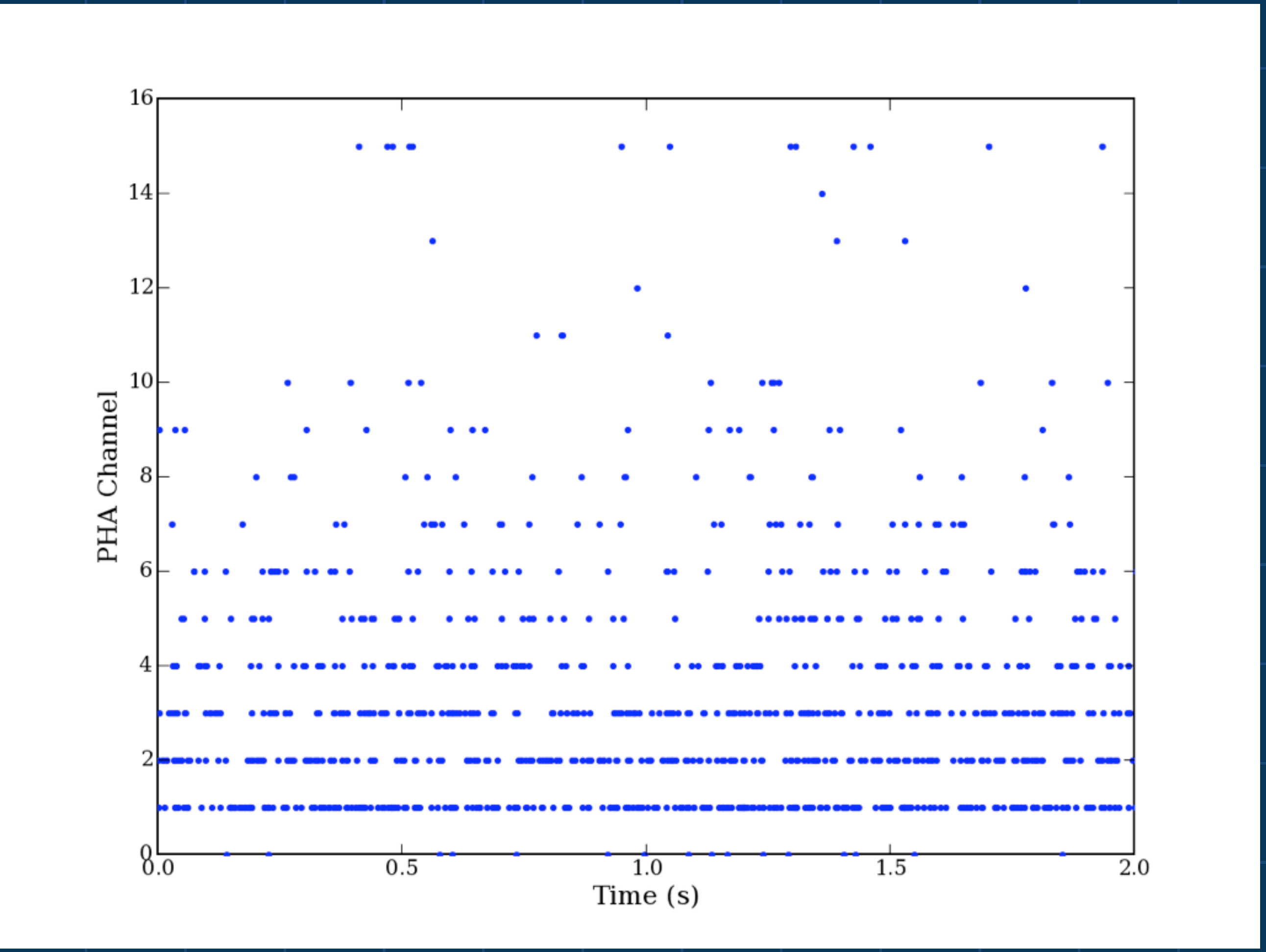
Time Domain Astronomy

- Astronomy is an *observational*, not *experimental* science
- Mostly done by characterizing the electromagnetic field impinging on Earth with a few exceptions (cosmic rays, neutrinos, gravitational waves)
- The EM field can be characterized by intensity a function of: angle, energy (i.e. frequency), polarization, and time.
  - Here, we will focus on the time domain, in other words, source variability.

X-ray Timing

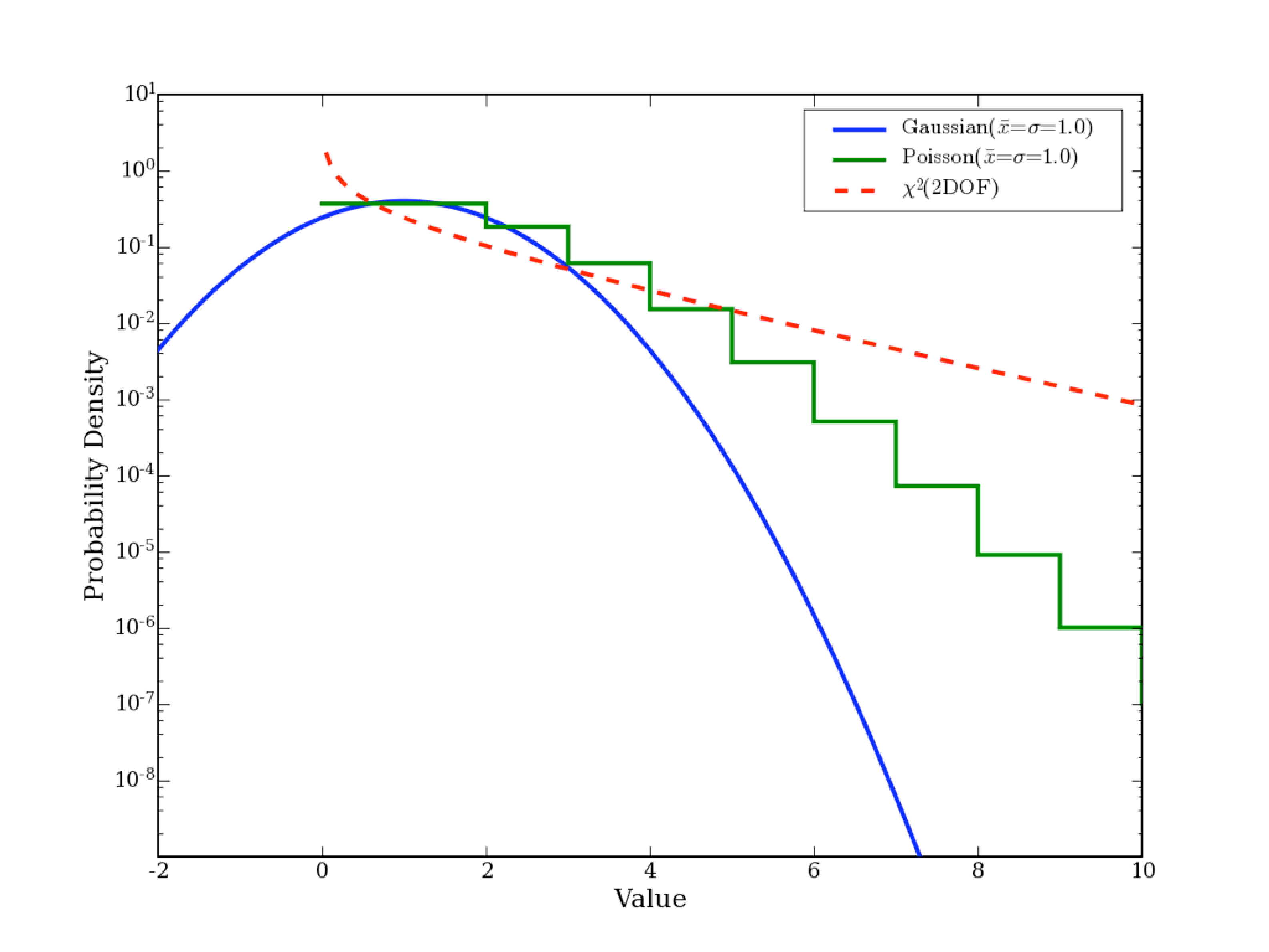
- In the X-ray band, detectors are sensitive to *individual* photons, which each carry significant energy ($E = h\nu$)
  - 1 keV = 1.6×10^{-9} erg = 2.24×10^{17} Hz = 1.24×10^{-7} cm
- Detectors can record the arrival time, energy, and direction of each photon (and perhaps polarization in the future)



2 seconds of raw data from GRS1915+105

Aside on Photon Statistics

- **Warning: Because we are counting individual photons, the relevant statistics are *Poisson*, not *Gaussian*.**



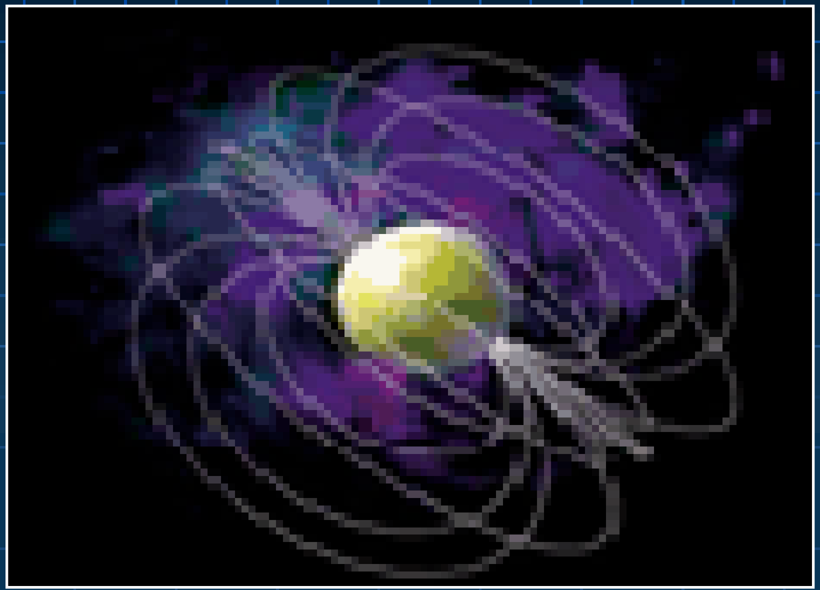
What Can We Learn From Timing?

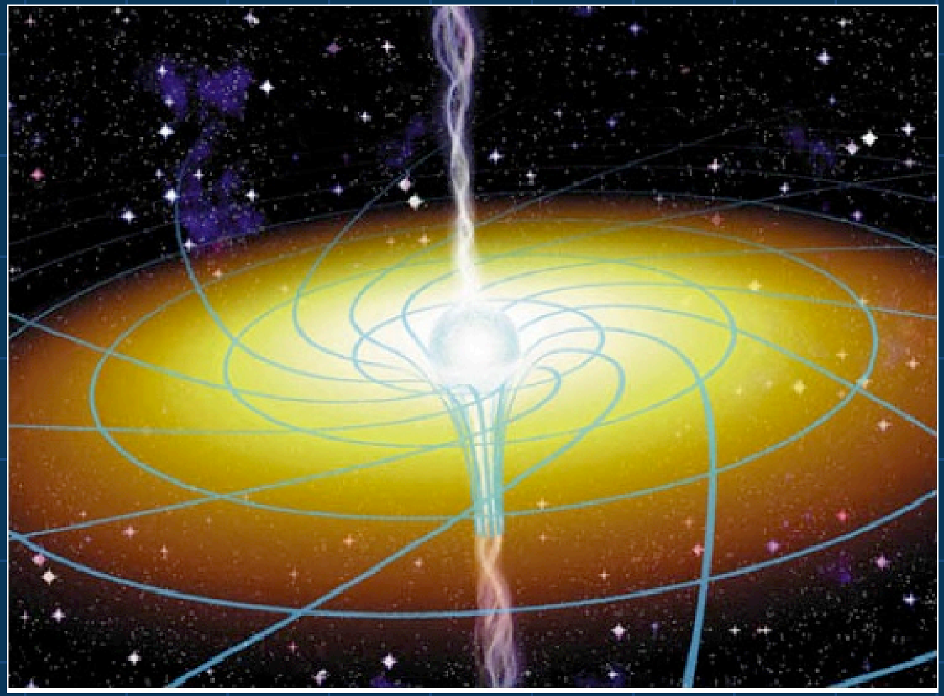
- Source variability probes geometry of the emitting region in a way spectra cannot
- Fastest time scales probe the smallest time size scales
  - Accretion dynamics near event horizon of BH or surface of NS, burning fronts propagating around NS, magnetic reconnection bursts on a magnetar
- Coherent pulsations allow extremely precise measurements
  - Orbital period and evolution, accretion torques, rotational glitches

Rotational Periods:

ms - s for NS/WD

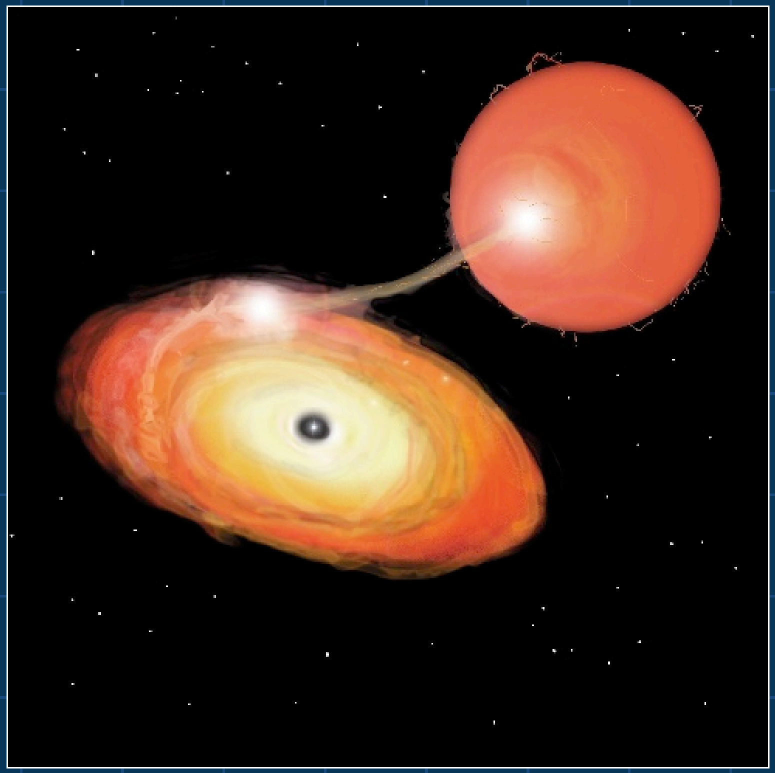
hr - days for Stars





Accretion Time Scales:

Dynamical, Thermal, & Viscous Time Scales
(e.g. QPOs, outburst timescales)
ms – days for NS/BHC
minutes – years for AGN



Orbital Time Scales:

minutes to days for NS/BHC
Suber-orbital periods:
weeks – months

X-ray Bursts & Superbursts

Characteristic Time Scales

$\tau \geq R/v, \; v \leq c, \; R \geq 2GM/c2$

- AGN ($10^8\,M_\odot$) $\Rightarrow \tau > 1000$ s
- Black Hole ($10\,M_\odot$) $\Rightarrow \tau > 100\,\mu s$
- Neutron Star ($1.4\,M_\odot$) $\Rightarrow \tau > 15\,\mu s$

These are the fastest achievable time scales. In reality, there is variability on a range of time scales.

Software Tools

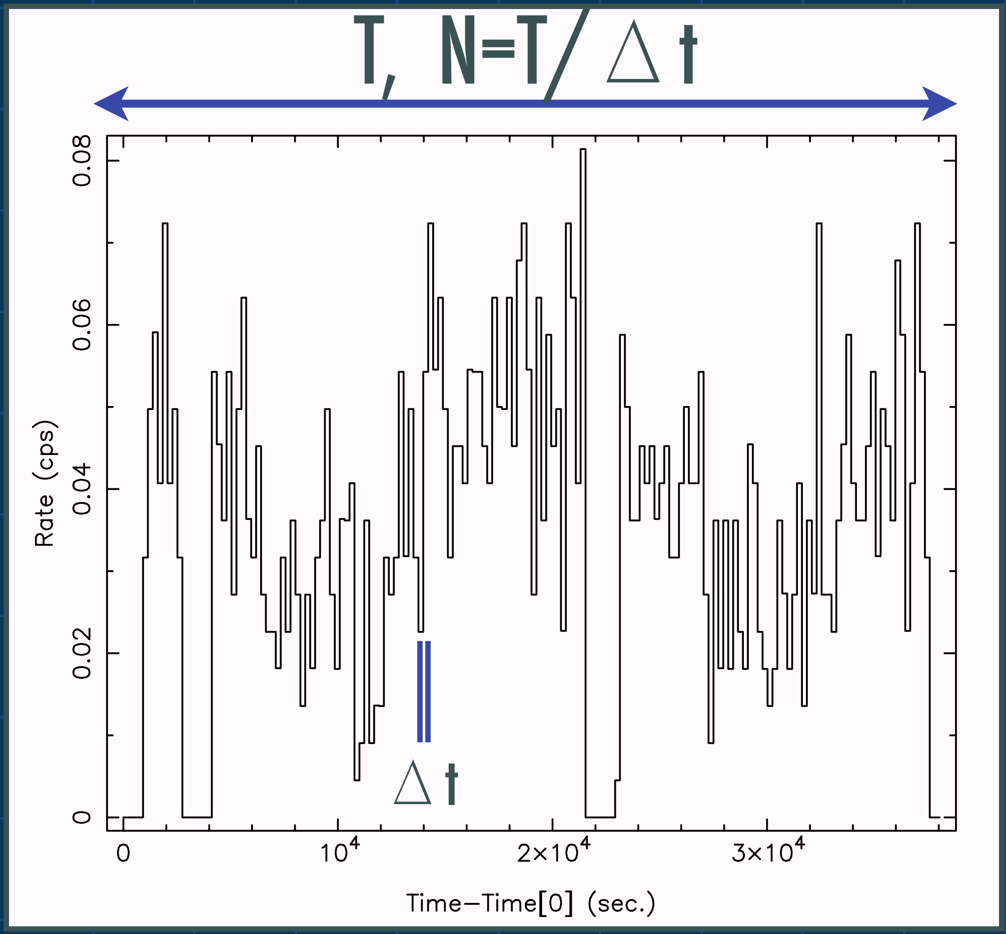
- HEASoft (FTOOLS)
  - Distributed by NASA’s HEASARC http://heasarc.gsfc.nasa.gov/docs/software/lheasoft/
  - Supports many mission formats (RXTE, Swift, etc...) and generic FITS files
- SITAR <http://space.mit.edu/CXC/analysis/SITAR>
  - Being developed by Mike Nowak
- Or, “roll your own” as many people do
  - Custom C or FORTRAN code
  - IDL or MATLAB
  - Python + SciPy&Matplotlib

Simplest Tool: A Lightcurve

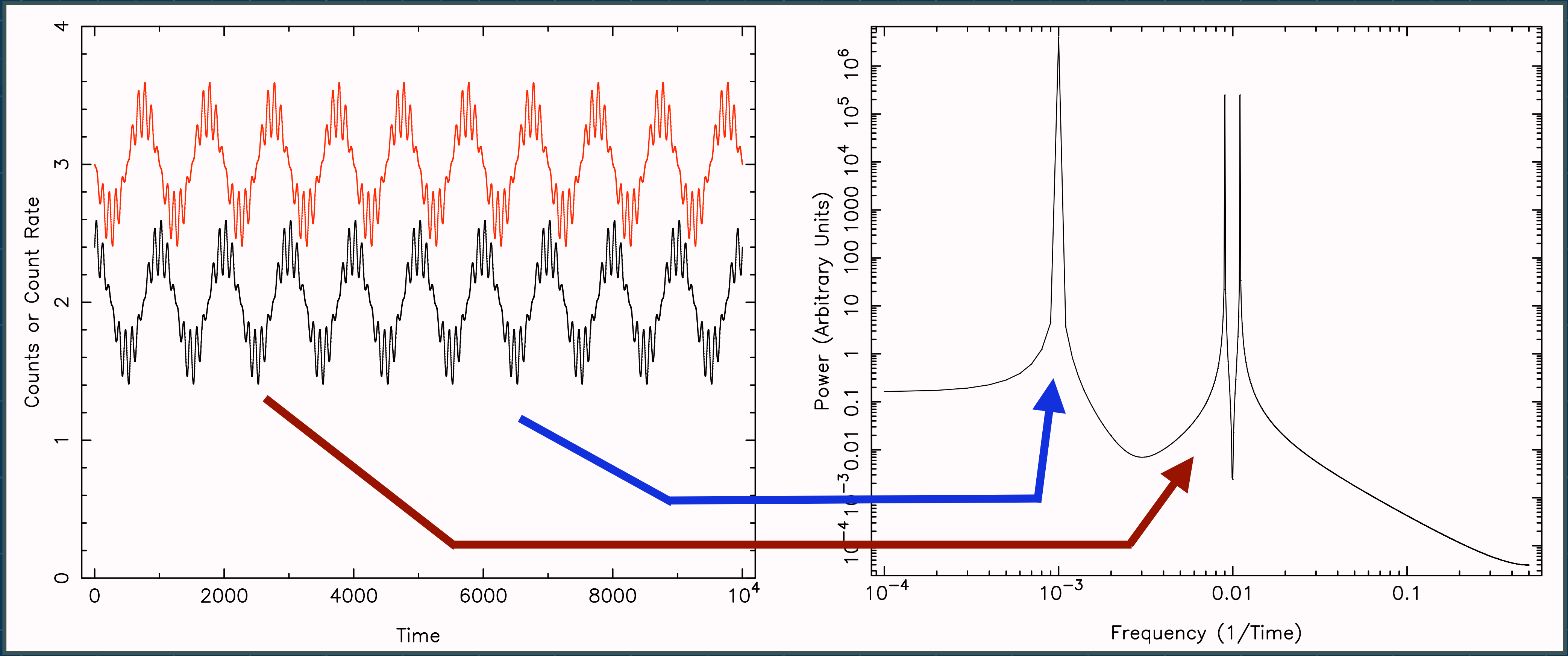
- **Select photons from an energy range of interest and "bin" them into evenly spaced time bins with N_i counts/bin**
- ***Tip: Always choose integer multiple of "natural" time unit for binning***
- **Don't bin more than you have to – save it for subsequent analysis**
- **Be careful to normalize by exposure time**
- **Once you convert from counts/bin to rates or subtract any background or DC component, error is no longer sqrt(N_i)**

Length & Binning Determine Limits

- Lowest Frequency: $f_{long} = 1/T$
- Highest Frequency: Nyquist Frequency, $f_{Nyq} = 1/(2\Delta t)$
- Basic Question, is the variance: $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$ greater than expected from Poisson noise?
- σ = Root Mean Square Variability



Fourier Transform Methods



- The workhorse of the timing world
- Describes how variability power is distributed as a function of frequency

Fourier Transform Definition

$$X_j \equiv \sum_{k=0}^{N-1} x_k \exp(2\pi ijk/N) \quad , \quad j = [-N/2, \ldots, 0, \ldots, N/2]$$

- **A Fourier Transform decomposes a time series into "sine waves" of different frequencies**
- **Power Density Spectrum (PDS) is the squared Fourier amplitude, properly normalized**
  - **Lightcurve with N bins, comprised of counts, x_i, becomes power spectrum, with $N/2+1$ independent amplitudes**
    - Discarding phases throws out information ⇒ power spectra are not unique!
  - **Know Your Normalization!!! Various FFT Routines Have Different Ones! (FTOOLS routine `powspec` gives you a choice)**
    - "One-sided" Leahy (mean power = 2): $P_j = 2|X_j|^2/N_{ph}$
    - "One-sided" (RMS/mean)^2/Hz: $P_j = 2|X_j|^2/(N_{ph}\times <\mathrm{Rate}>)$

Useful Theorems

- **Fourier Transform is a linear transform**

$$ax(t) \Leftrightarrow aX(f)$$

- **Real-valued data:**

$$x_k \in \Re \Rightarrow X_{N-j} = X_j^* \text{ where } j \in [1, N/2-1]$$

- **Parseval's Theorem**

$$\sum_{k=0}^{N-1} |x_k|^2 = \frac{1}{N} \sum_{j=0}^{N-1} |X_j|^2$$

- **Shift**

$$x(t-t_0) \Leftrightarrow X(f)e^{2\pi i f t_0}$$

FAST Fourier Transforms

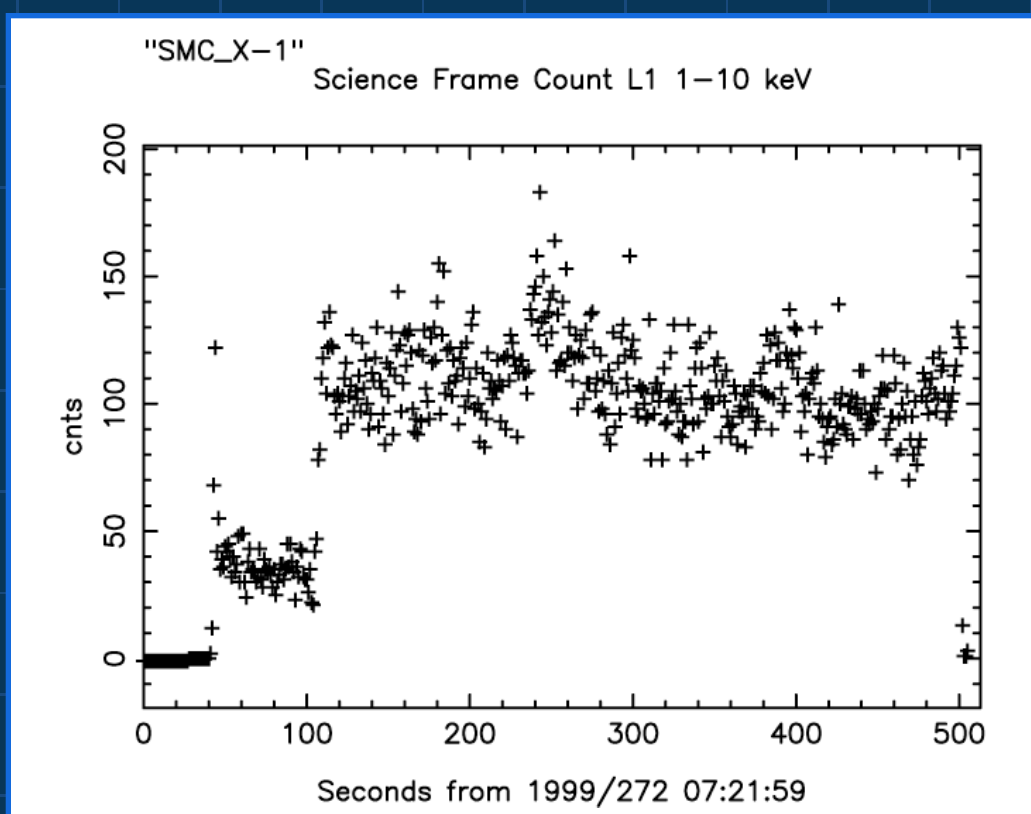
- FFT algorithm (Cooley & Tukey 1965) transformed problem from $O[N^2]$ to $O[N \log_2(N)]$ which *greatly* increased the usefulness of Fourier techniques
- Current state-of-the art is the FFTW ("Fastest Fourier Transform in the West") library by Frigo & Johnson (MIT)
  - Many FFTs require or strongly prefer $N=2^n$, but FFTW works well with any small prime factors and still works even with N=prime.
  - It is highly portable (Linux/Mac/Windows/...) and is close to the fastest possible FFT on every platform with no special effort.
  - Get it! <http://www.fftw.org>

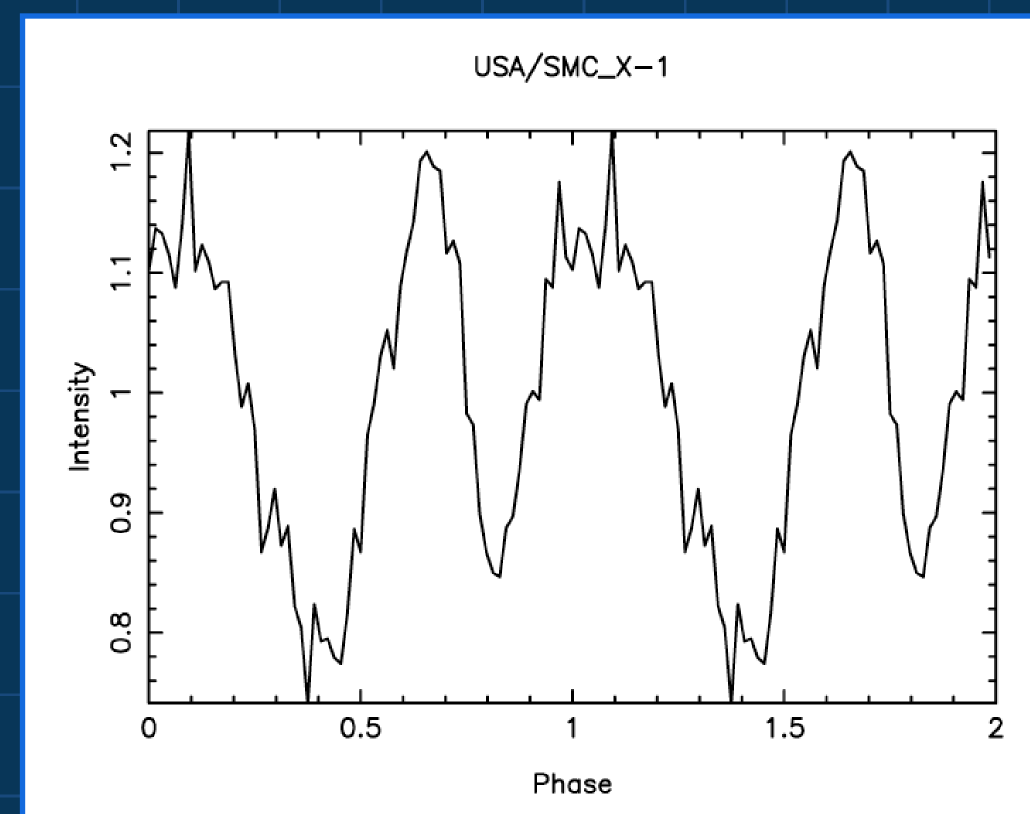
Coherent Signals

- Much analysis involves “coherent” signals, i.e. periodic signals whose phase is constant over the relevant duration
  - Or, equivalently, where a time transformation (sometimes called a “timing model”) can be determined that makes the signal coherent
- Examples:
  - Pulses from rotating pulsars
  - Orbital modulation or eclipses
  - Precession periods

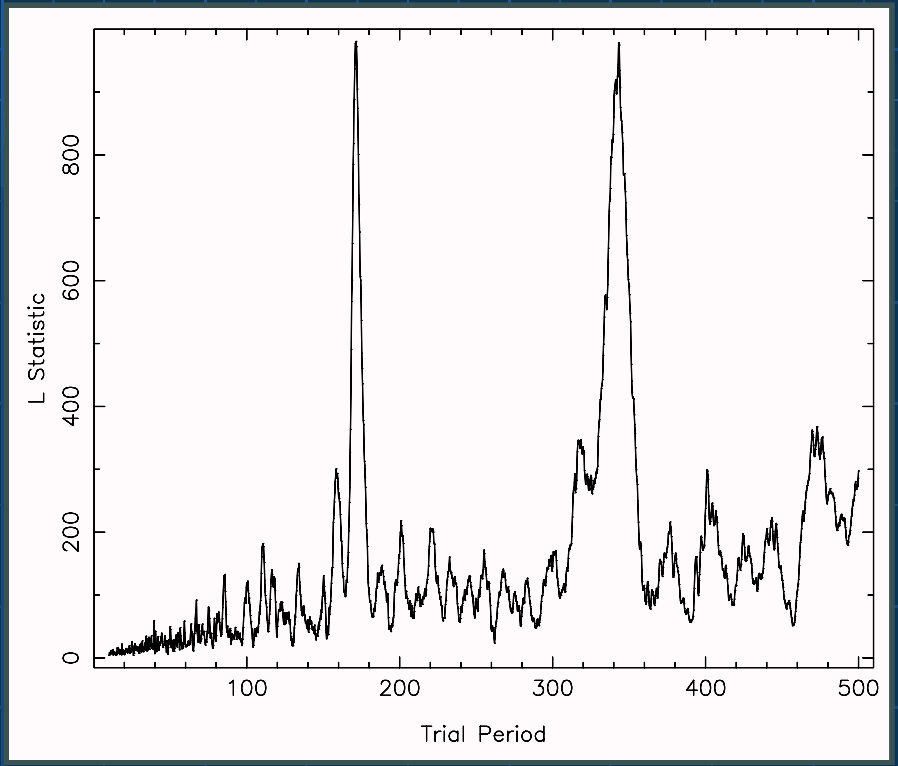
Epoch Folding

- Bin photons according to phase with respect to a *known* period P (or a more complicated timing model)
- Significance of variability at that period can be assessed by doing a χ^2 test against a null hypothesis of constant rate.



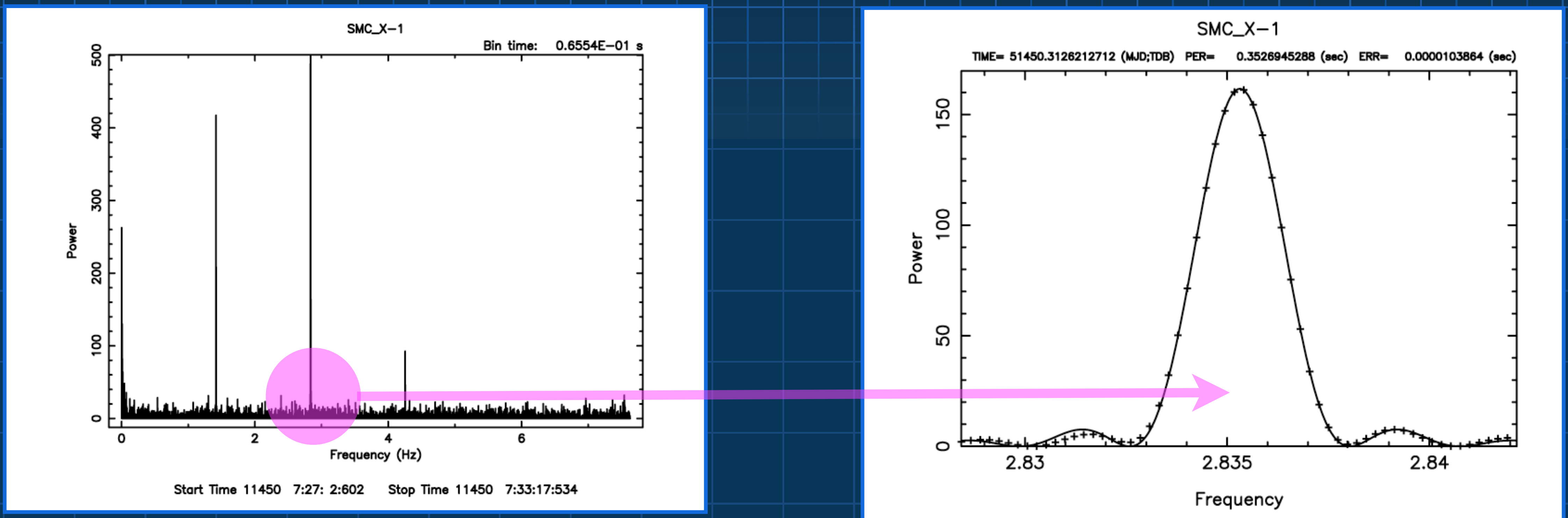


Epoch Folding Searches



- Perform epoch folding at a large number of *trial periods*, and look for trials with large χ^2
- Good for non-sinusoidal variations, and when there are data gaps or complicated window functions
- Can be slow to explore a large range of periods
  - Requires N_{ph}*N_{per} operations: `fmod(t_ph,P)`

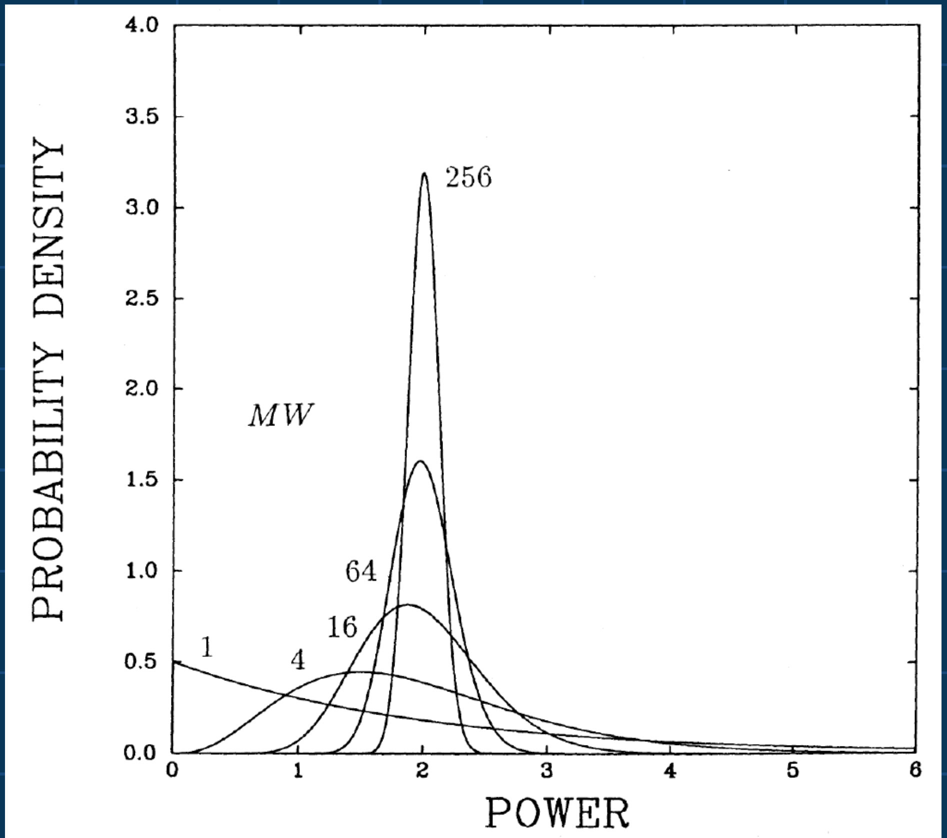
FFT Searches



- Pros
  - MUCH faster than epoch folding searches in many cases
  - Searches all possible frequencies simultaneously
- Cons
  - Potentially large memory requirements
  - Requires harmonic summing for non-sinusoidal signals

Statistics of Power Spectra

- How do you determine the significance of peaks found in power spectra?
- Distribution of P_k is χ^2/MW with $2MW$ D.O.F., where MW is the number of power spectra summed
- So, just compute the probability of a false occurrence: $\Pr(P_k >$ thresh$)$
  - Number of trials is critical!
  - Distribution has a LONG tail!



$$\mathrm{PDF}[\chi_r^2(x)] = \frac{2^{-r/2} e^{-x/2} x^{\frac{r}{2}-1}}{\Gamma\left(\frac{r}{2}\right)}$$

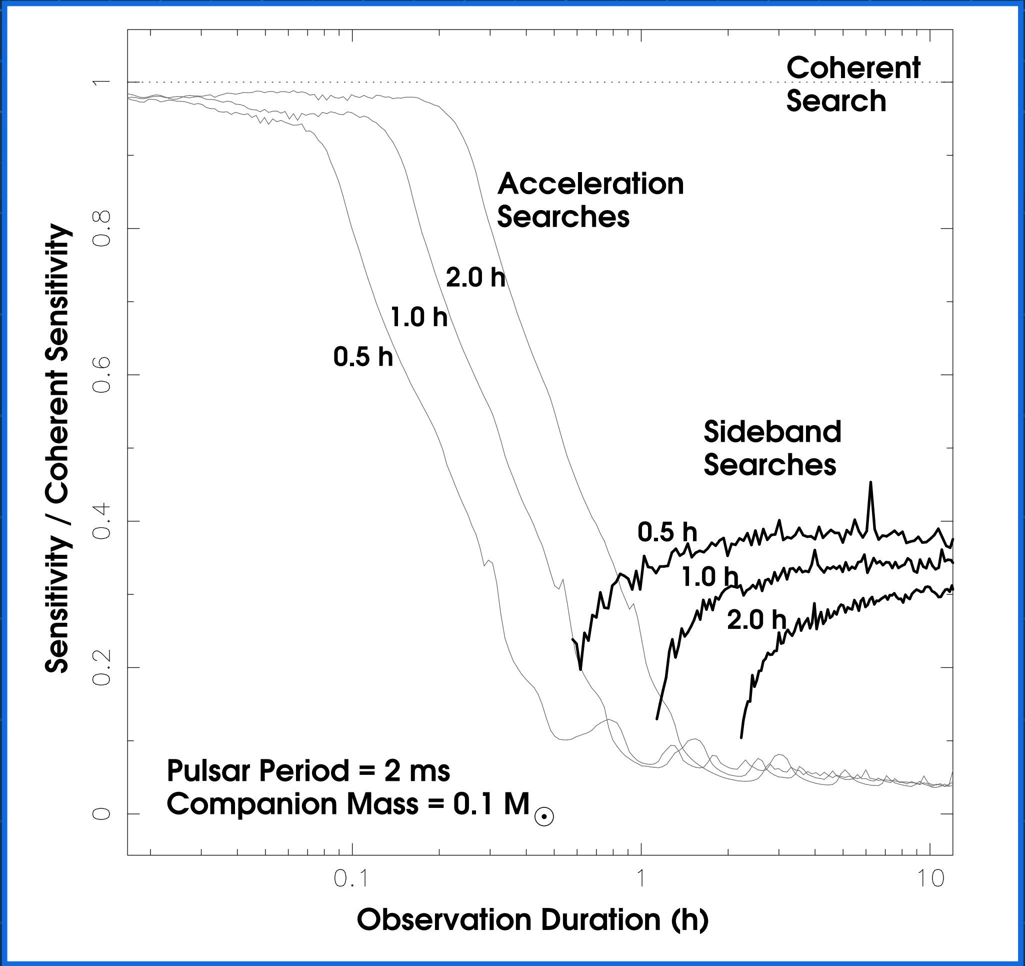
Decoherence

- FFT and simple epoch-folding searches require the single be coherent throughout the interval being considered
- But, this might not be the case because of:
  - Orbital Doppler shifts from a binary system
  - Intrinsic period derivative of the source
  - Satellite or Earth motion that isn't fully compensated for
- Searching still possible with several techniques

Acceleration Searches

- Attempt to transform the time series into a frame where the signal is coherent
- Stretch the time series according to a set of trial accelerations, or matched filter in the Fourier domain
  - Assumes constant acceleration during observation
- Only works when higher order terms can be ignored (e.g. when $T_{obs} < P_{orb}/10$)
  - Wood et al. (1991, ApJ, 379, 295); Vaughan et al. (1994, ApJ, 435, 362)
  - Ransom, Eikenberry, & Middleditch (2002, AJ, 124, 1788)

Sideband (Phase-Modulation) Search



- When $T_{obs} > P_{orb}$, the response to the FFT of a sinusoidal signal is analytically calculable as a Bessel function
  - Ransom et al. 2003 ApJ, 589, 911
- Perform matched filter in the Fourier domain
- Recovers substantial fraction of fully coherent search sensitivity at a tiny fraction of the computational cost!

Pulsar Timing

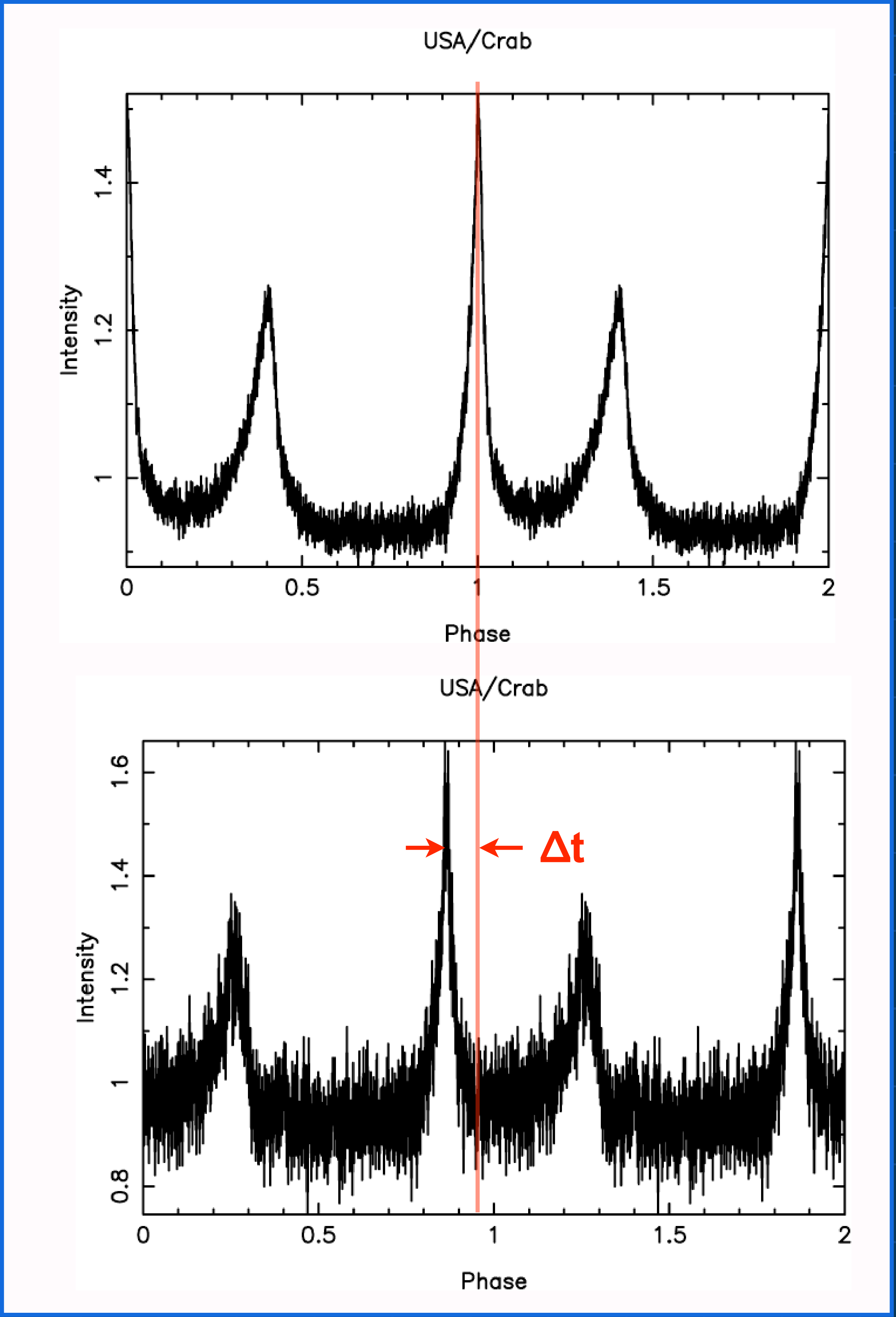
- Coherent timing over long time baselines is very powerful and precise since *every* cycle is accounted for
- Goal: To determine a *timing model* that accounts for all of the observed pulse arrival times (TOAs)
- Parameters that can be determined:
  - Spin (ν, ν', ... ⇒ torques, magnetic fields, ages)
  - Orbital (P_{orb}, T_0, e, ω, $a_x \sin i$, GR terms)
  - Positional (α, δ, π, proper motion)

Measuring a TOA

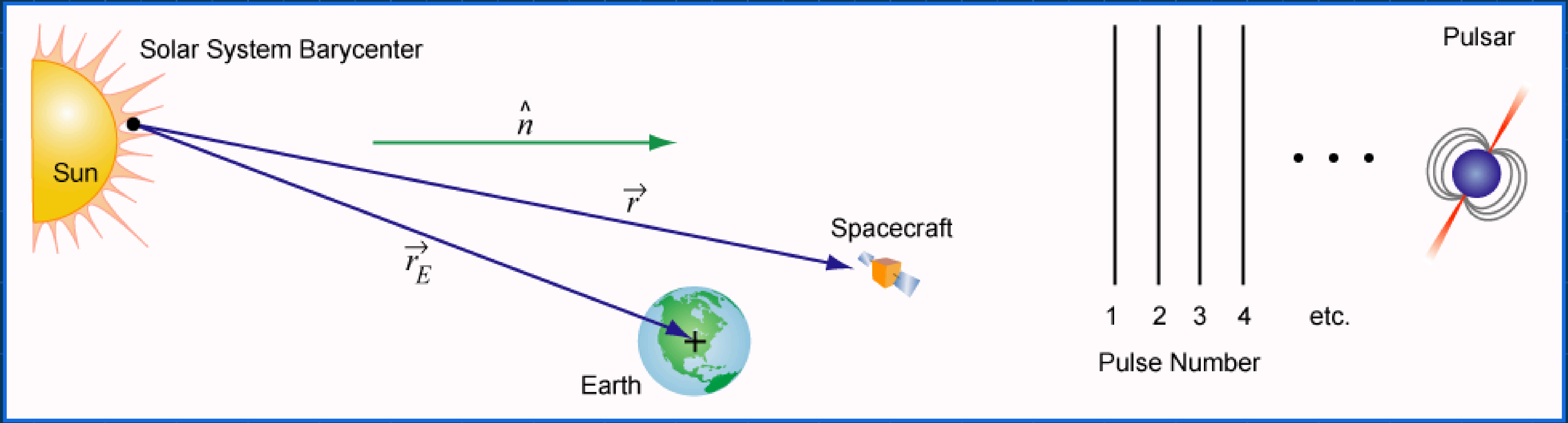
- **Measure phase shift between measured TOA and a template profile**
  - Application of the FFT shift theorem (and linearity)

$$x(t - t_0) \Leftrightarrow X(f)e^{2\pi i f t_0}$$

- **TOA = T_{obs} + Δt**



Barycentering TOAs

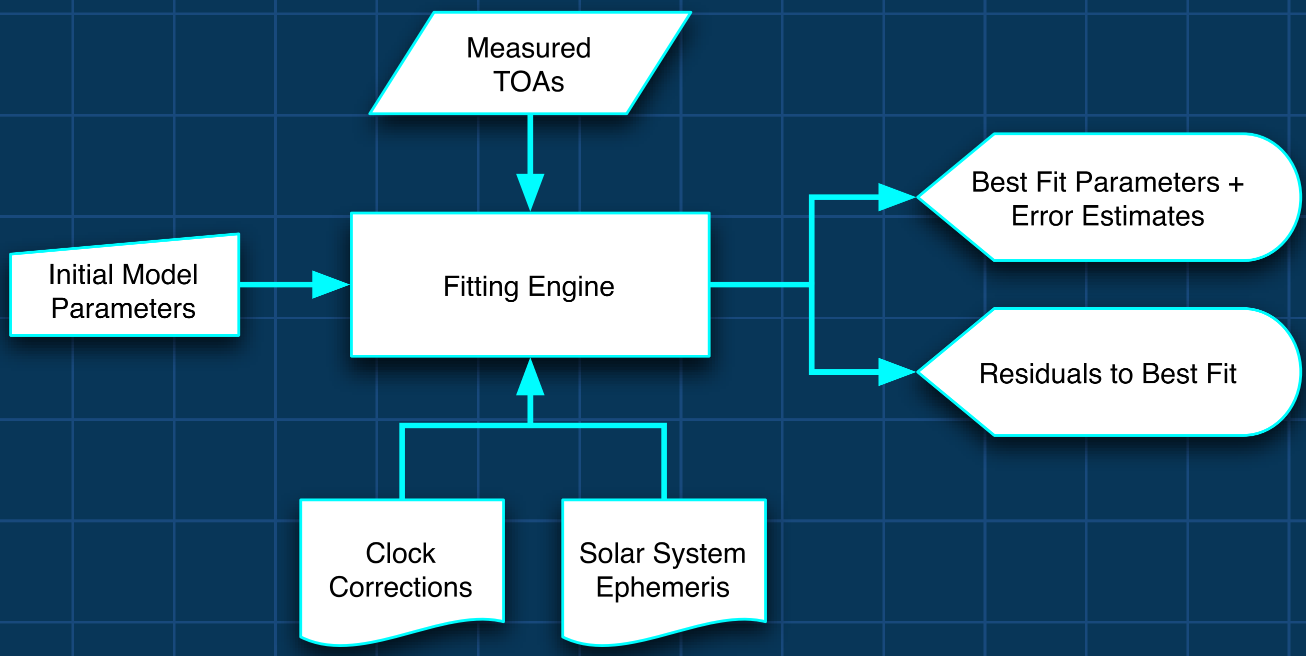


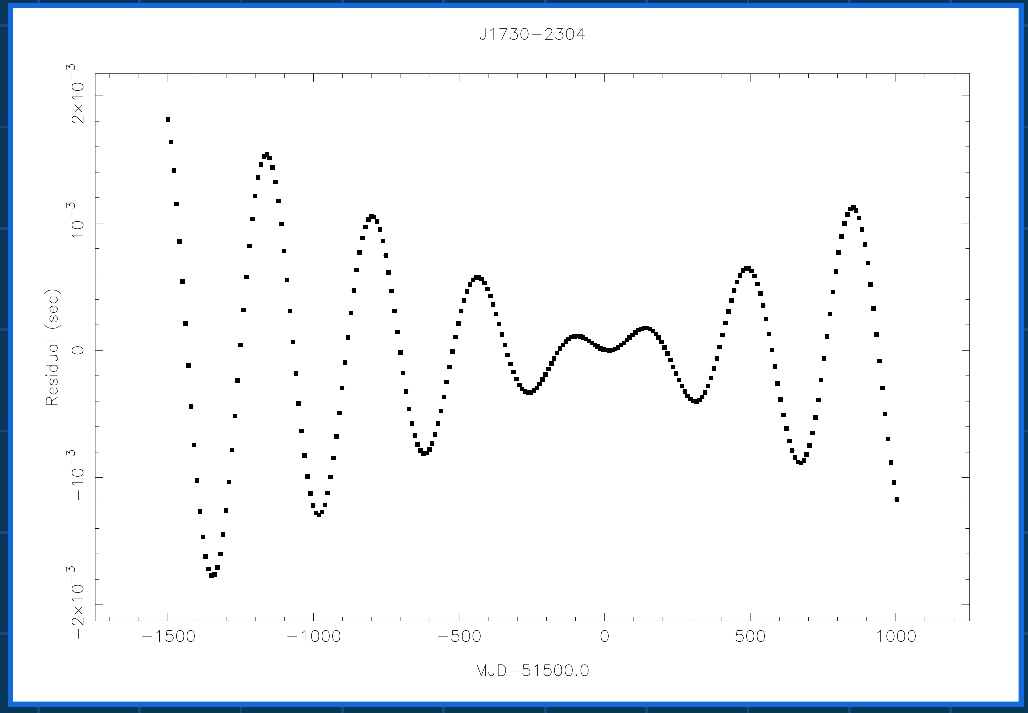
- Arrival times at Earth or spacecraft must be converted to a nearly inertial frame before attempting to fit a simple timing model
- Remove effects of observer velocity and relativistic clock effects
- Convenient frame is the Solar System Barycenter

Fitting TOAs to a Timing Model

$$\phi(t) = \phi(0) + \nu t + \frac{1}{2}\dot{\nu}t^2 + \frac{1}{6}\ddot{\nu}t^3 + \ldots$$

Full model can include spin, astrometric, binary, and other parameters.





- **Goal: Find parameter values that minimize the residuals between the data and the model**

Tools for Fitting Timing Models

- **Tempo < http://pulsar.princeton.edu/tempo/>**
  - Developed by Princeton and ATNF over 30+ years
  - Well tested and heavily used
  - Based on TDB time system
  - But, nearly undocumented, archaic FORTRAN code
- **Tempo2 < http://www.atnf.csiro.au/research/pulsar/tempo2/>**
  - Developed at ATNF recently (still in beta test)
  - Based on TCB time system (coordinate time based on SI second)
  - Well documented, modern C code, uses `long double` (128 bit) throughout
  - Easy plug-in architecture to extend capabilities
  - But, not well tested, still in development

Time Systems

TAI = Atomic time based on the SI second

UT1 = Time based on rotation of the Earth

UTC = TAI + "leap seconds" to stay close to UT1

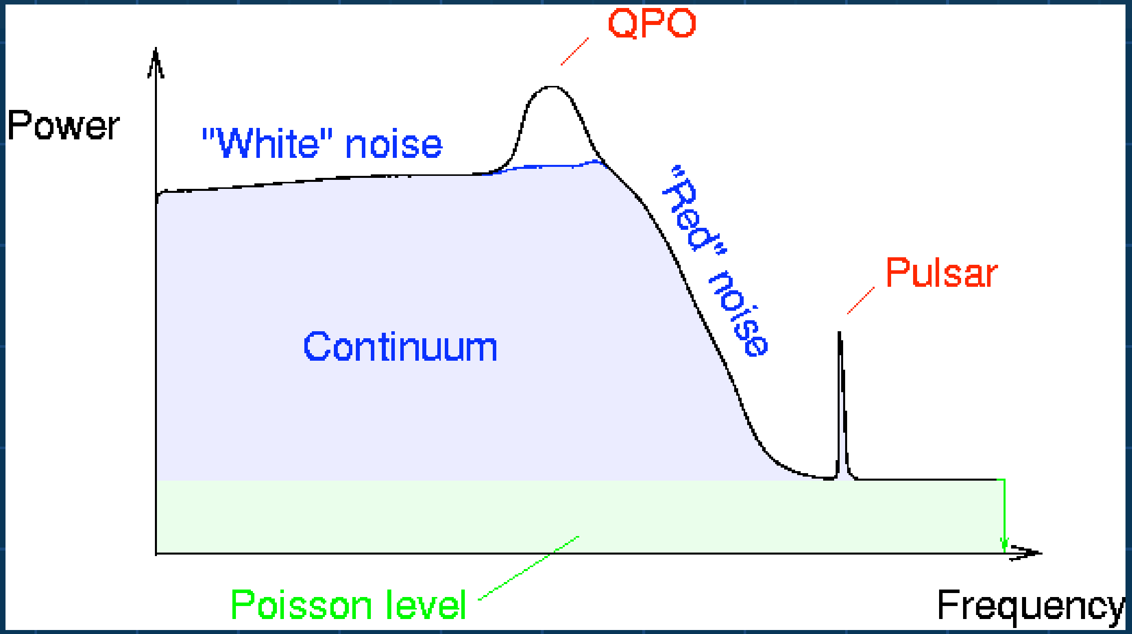
TT = TAI + 32.184 s

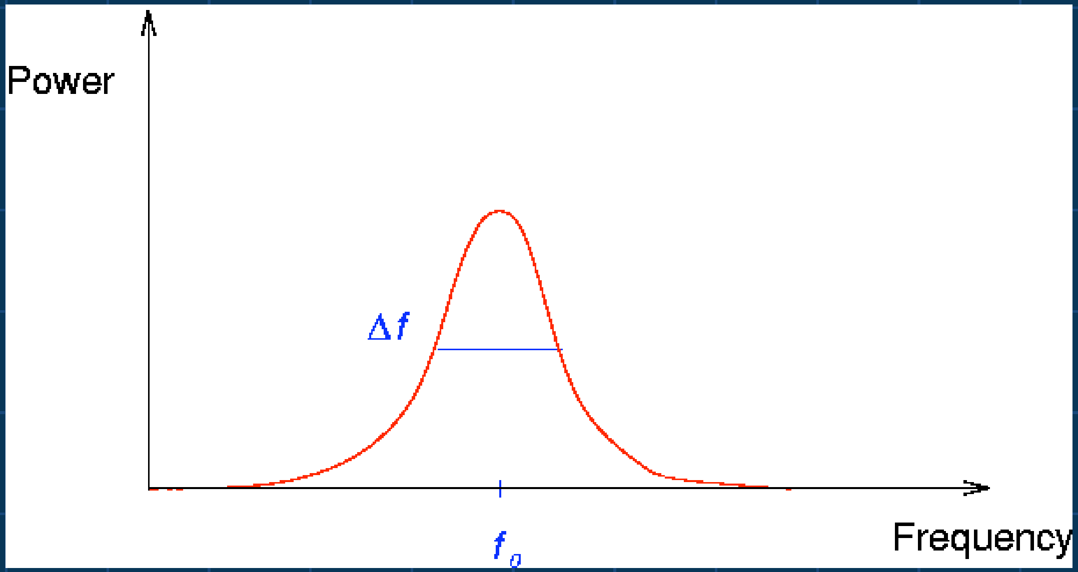
TDB = TT + periodic terms to be uniform at SSB

TCB = Coordinate time at SSB, based on SI second

Aperiodic Variability

- **The broadband power spectrum can characterize:**
  - **Total (excess) variability**
  - **Power spectral slopes and breaks (special time scales)**
  - **Quasiperiodic oscillations (QPOs)**
    - Random walks in phase or frequency
    - Finite lifetime of processes
    - Amplitude modulation

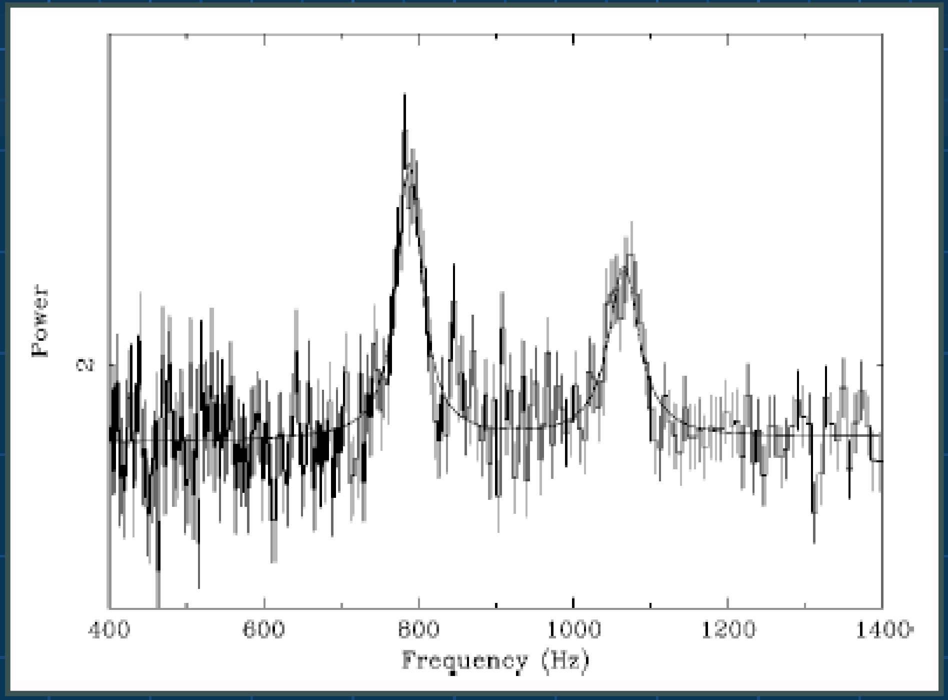




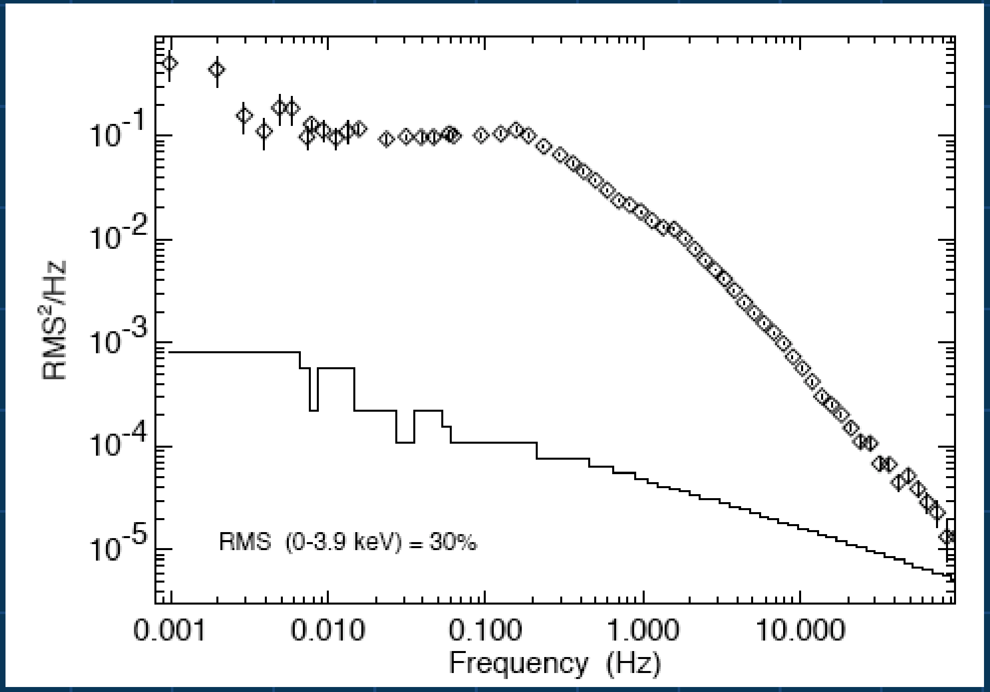
"Quality factor" Q = $f_0/\Delta f$

Rebinning and Averaging

- Single FFT bin is a terrible estimator of the PSD, because of the huge variance
- Making FFT longer doesn't help; just samples frequencies more finely
- Solutions:
  - Average adjacent frequency bins (often done logarithmically)
  - Average PSDs of multiple data segments



"Twin" kHz QPOs



"Band-Limited Noise"

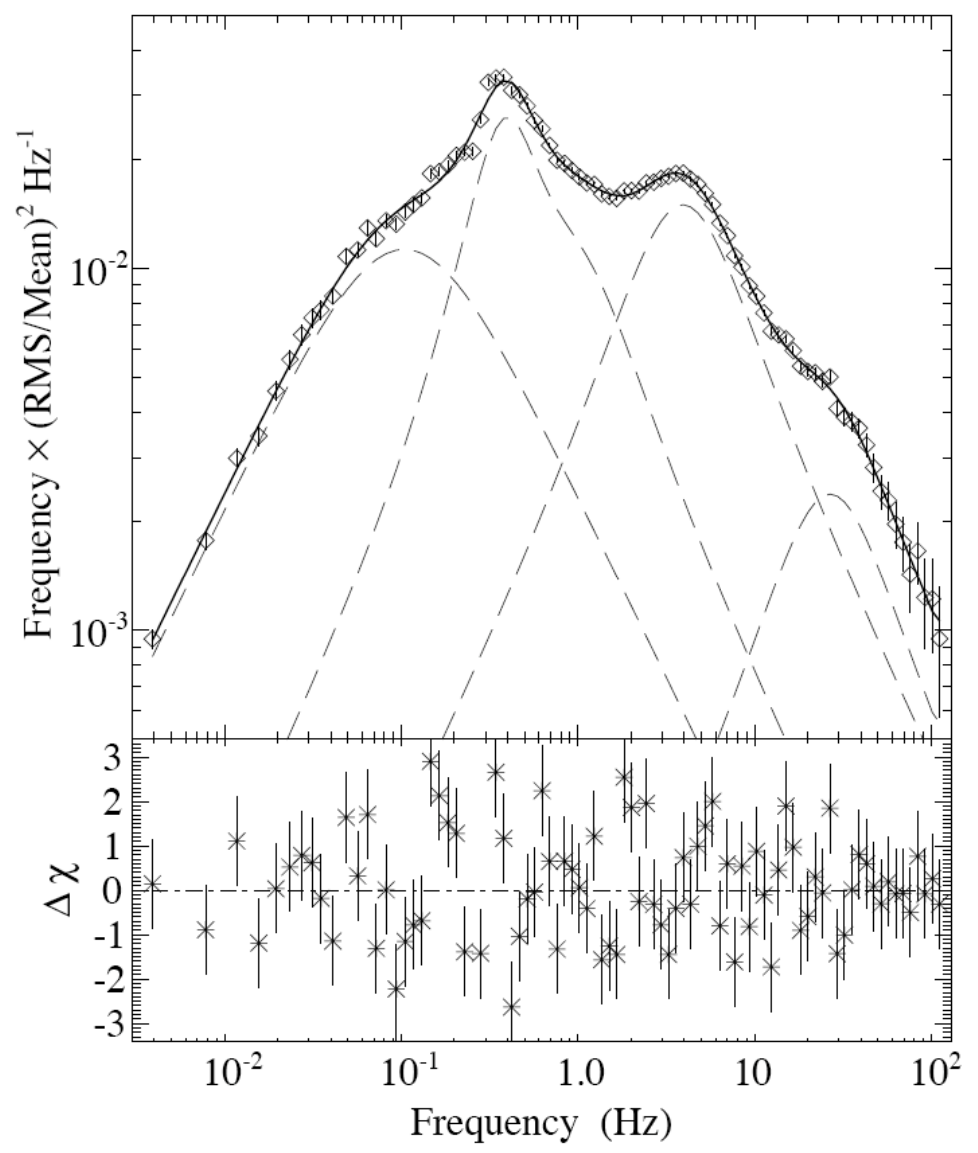
PSD Model Fitting

- After subtracting Poisson level, you can fit models

$$P'_j = (P_j - P_{\rm noise}) \pm P_j/\sqrt{N_{\rm avg}}$$

- Popular choice currently is a sum of Lorentzians
  - See Belloni, Psaltis, & van der Klis (2002, ApJ, 572, 392)

$$L(x) = \frac{\Gamma}{2\pi\left((x-x_0)^2+\frac{\Gamma^2}{4}\right)}$$



Dead Time Effects

- Detector "Deadtime" is when the detector can't detect events, either:
  - For a period of time after an event
    - Paralyzable (event during deadtime extends deadtime)
    - Non-paralyzable (events during deadtime have no effect)
  - Or, for a detector reason, such as readout intervals
- Deadtime modifies the power spectrum of Poisson noise from the expected $P_{Leahy} = 2$ (usually to something < 2)
  - See: Zhang et al. (1995, ApJ, 449, 930); Morgan et al. (1997, ApJ, 482, 993), Nowak et al.(1999, ApJ, 510, 874)

Advanced Topic: Unevenly Sampled Data

- Lomb Periodigram
- Bayesian Methods
- Wavelets

Review/Tips

- Coherent pulsation (e.g. pulsar) best done with no rebinning
- Pulsar timing is a powerful and precise tool
- QPO searches need to be done with *multiple* rebinning scales
- Beware of spurious signals introduced by:
  - Instrument (read times, clock periods, ...)
  - Dead time
  - Spacecraft orbit (background rate variations)
  - Diurnal/Annual effects

Proposal Estimates

- Detecting broad band noise (or QPO) at the n_σ confidence level
  - For broad band timing, you win more with rate than time

$$\mathrm{RMS}^2_{\mathrm{limit}} \approx 2n_\sigma \sqrt{\Delta f} / \sqrt{\mathrm{Rate}^2 \times T_{\mathrm{total}}}$$

- Detecting coherent pulsations

$$f_p^{\mathrm{limit}} = 4n_\sigma / (\mathrm{Rate} \times \mathrm{Time})$$

References for Further Reading

- van der Klis, M. 1989, “Fourier Techinques in X-ray Timing”, in *Timing Neutron Stars*, NATO ASI 282, eds. Ögelman & van den Heuvel, Kluwer
  - Superb overview of spectral techniques!
- Press et al., “*Numerical Recipes*”
  - Clear, brief discussions of many numerical topics
- Leahy et al. 1983, ApJ, 266, p. 160
  - FFT & PSD Statistics
- Leahy et al. 1983, ApJ, 272, p. 256
  - Epoch Folding
- Davies 1990, MNRAS, 244, p. 93
  - Epoch Folding Statistics
- Vaughan et al. 1994, ApJ, 435, p. 362
  - Noise Statistics
- Nowak et al. 1999, ApJ, 510, 874
  - Timing tutorial + coherence techniques

Data Exercises

- Get a computer with HEASoft installed
  - Linux/Mac/Sun/OSF etc... (Windows only under Cygwin)
- Measure the pulsations from Sco X-1
- Find the 0.1 Hz QPO in XTE J1118+480