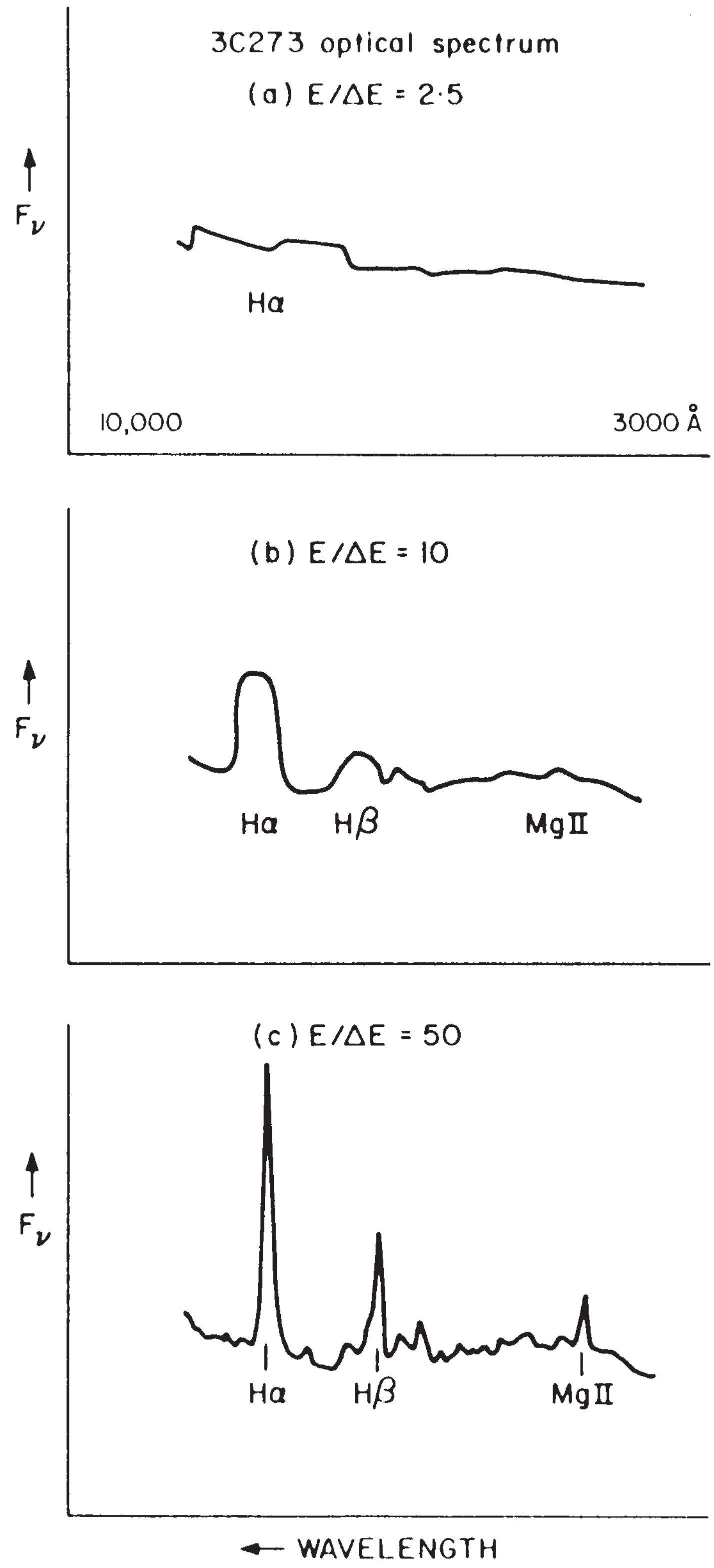
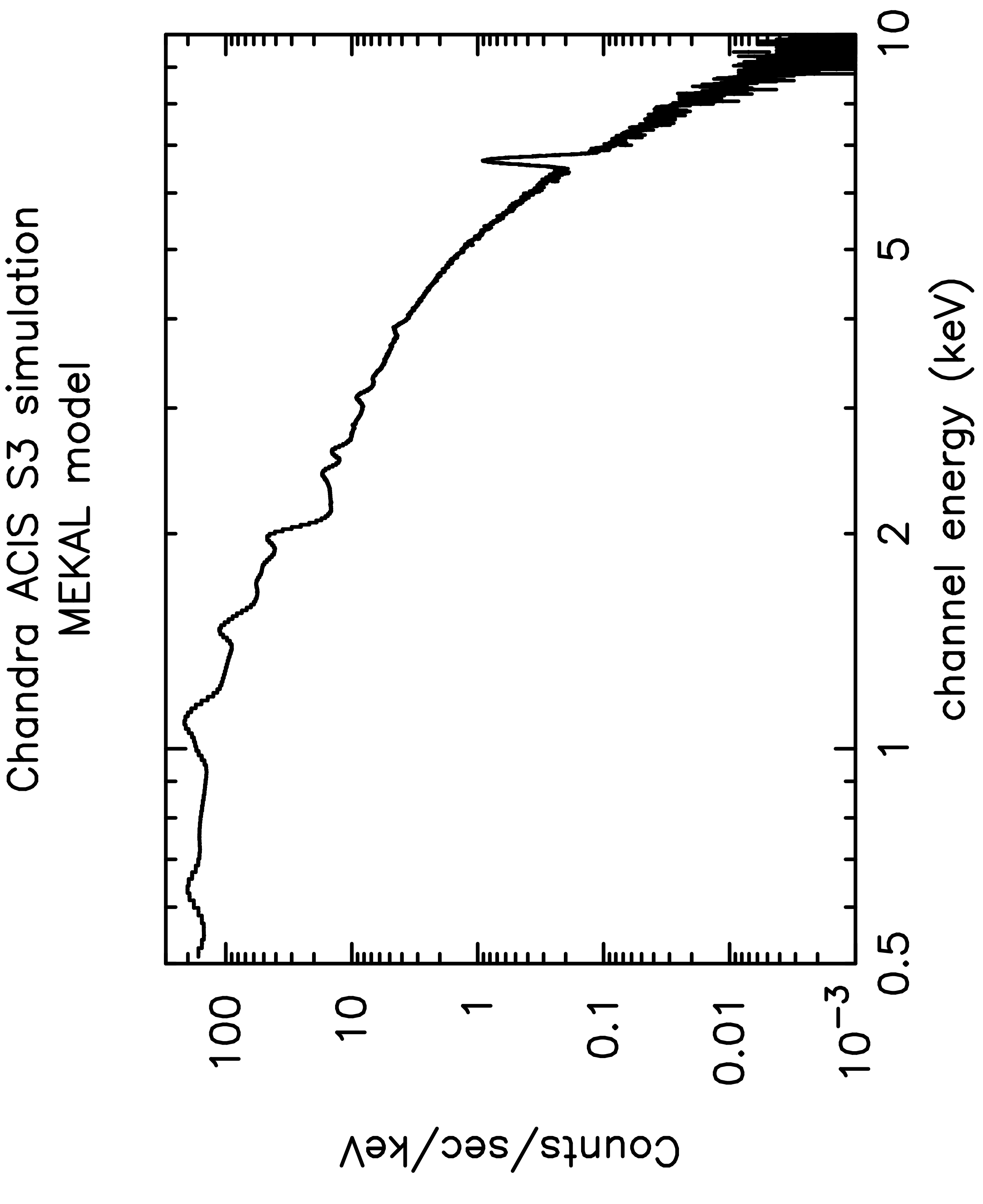
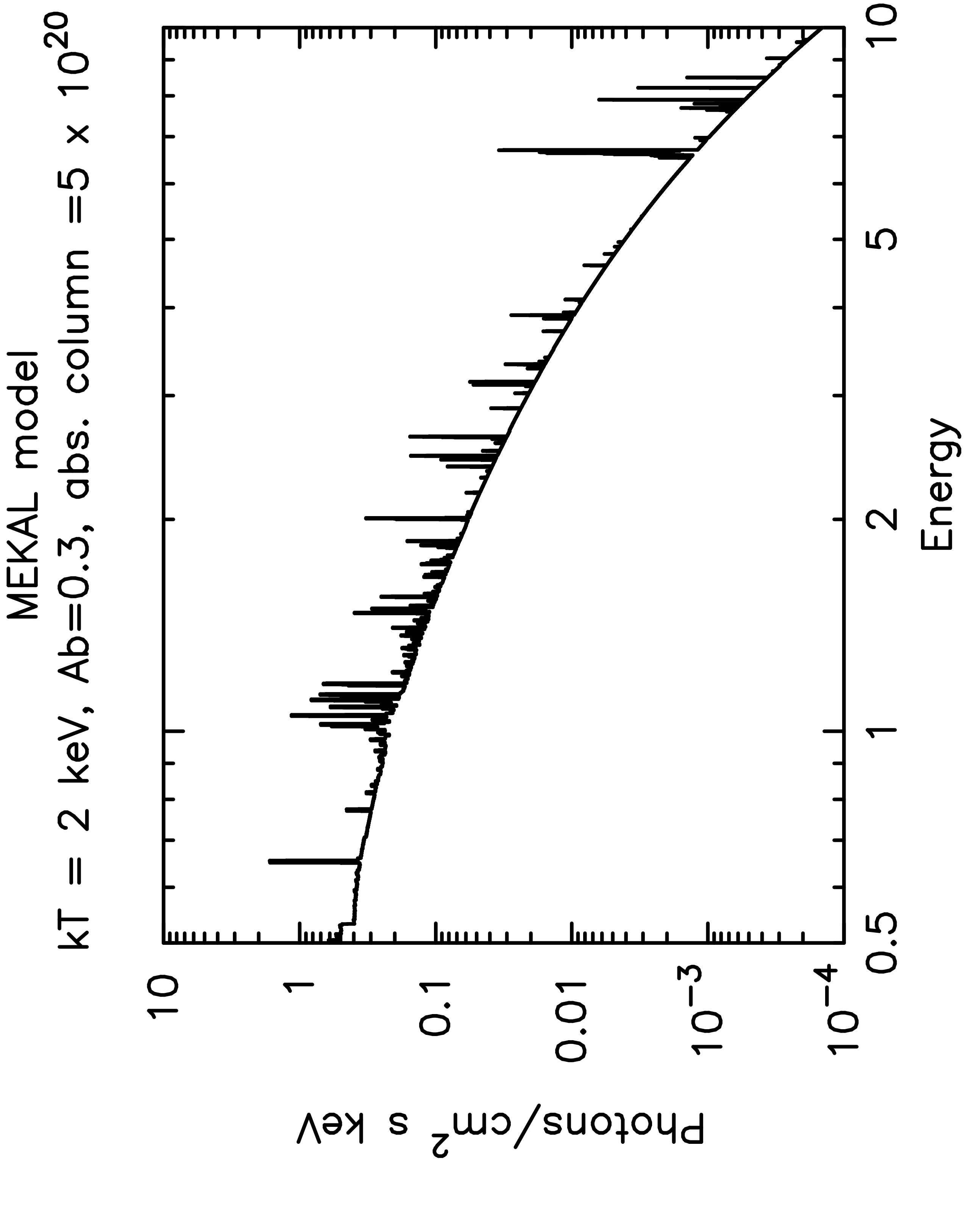
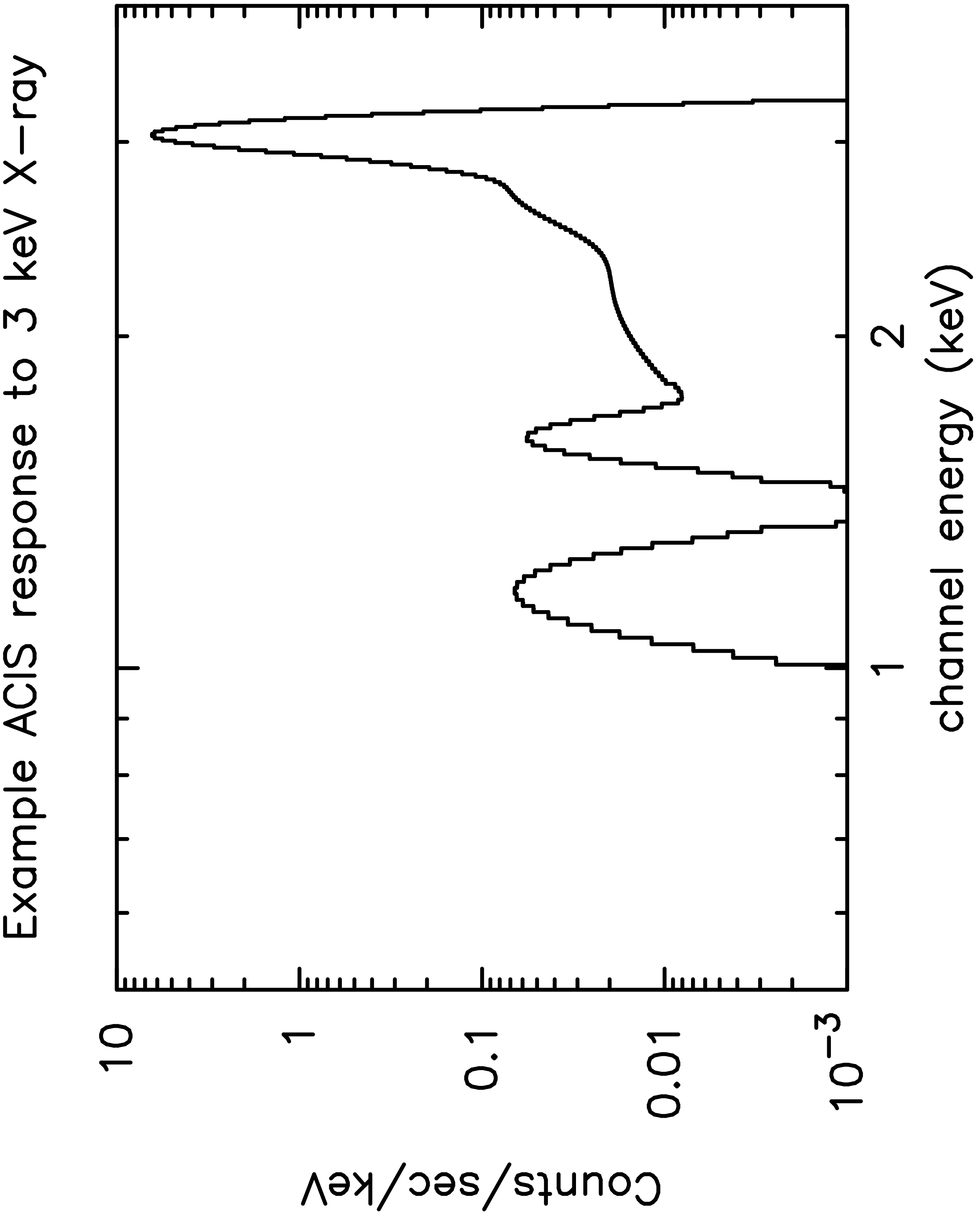


3C273 optical spectrum
(a) E/ΔE = 2·5
Fν
Hα
10,000
3000 Å
(b) E/ΔE = 10
Fν
Hα
Hβ
MgII
(c) E/ΔE = 50
Fν
Hα
Hβ
MgII
← WAVELENGTH


Chandra ACIS S3 simulation
MEKAL model
Counts/sec/keV
100
10
1
0.1
0.01
10^{-3}
0.5
1
2
5
10
channel energy (keV)



MEKAL model
kT = 2 keV, Ab=0.3, abs. column =5 x 10^20
Photons/cm^2 s keV
Energy
10
1
0.1
0.01
10^-3
10^-4
0.5
1
2
5
10



Example ACIS response to 3 keV X−ray
Counts/sec/keV
10
1
0.1
0.01
10−3
channel energy (keV)
1
2

The “Remote Sensing Problem”

- **The matrix equation :**
  $$\boldsymbol{Observation(i) = \Sigma_j\, Response(i,j) * Model(j)}$$
  where i is over channels and j is over energies is a discrete form of the Fredholm integral of the first kind. This occurs in many branches of science and medicine from CAT scans and MRI to geophysics to all wavelength bands in astronomy.

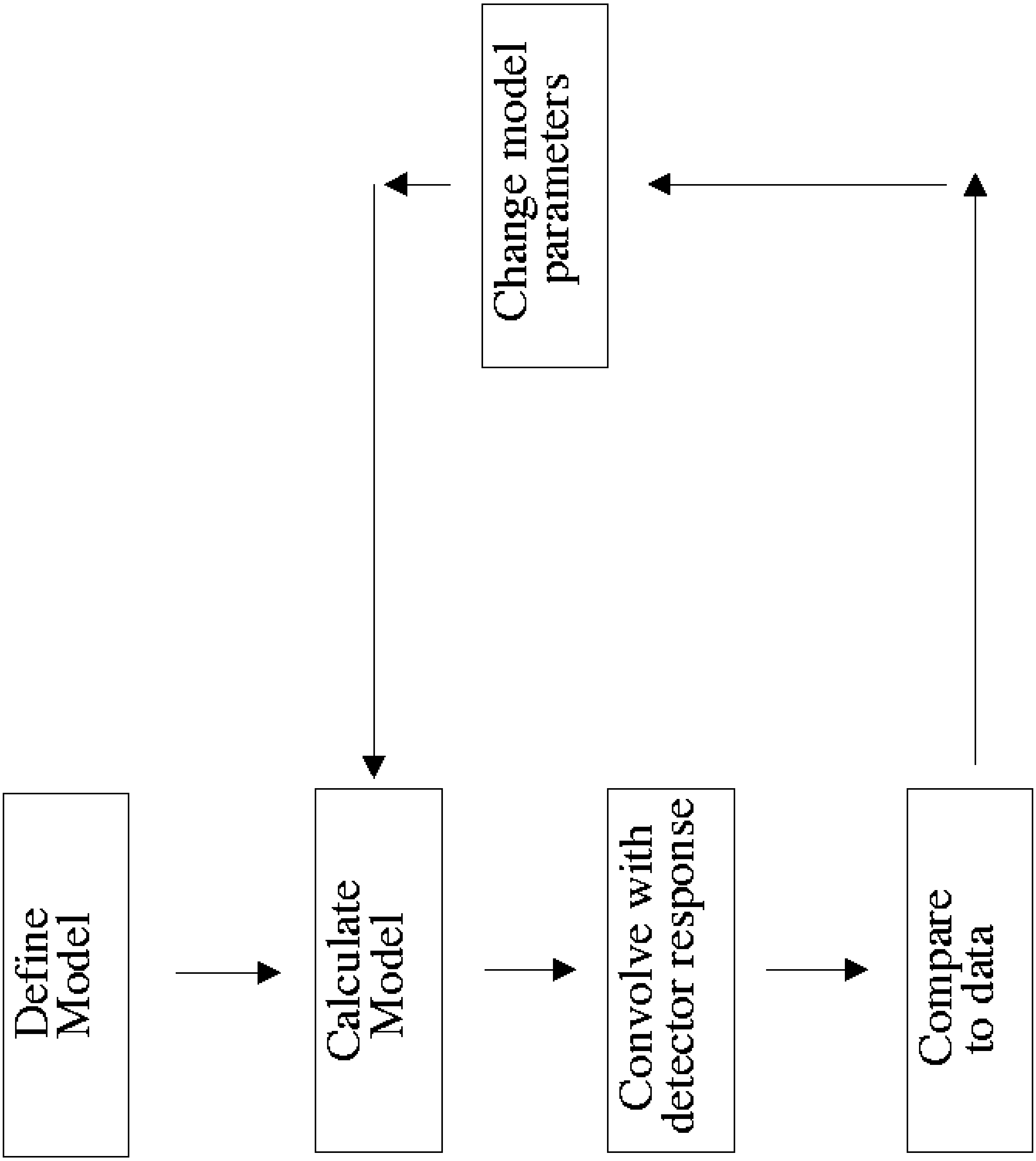
- **It is well-known that multiplying both sides of the equation by the inverse of the response matrix in order to recover the Model does not work. This can have multiple solutions and excessive sensitivity to small perturbations in the data.**

- **The Russian mathematician, Tikhonov, developed a solution in the early 1960s known as regularization. This stabilises the inversion basically by using some a priori constraint eg smoothness. A familiar example in astronomy is maximum entropy deconvolution.**

- **There have been sporadic attempts in X-ray astronomy to use variants of regularization but none have been particularly successful because a) it is not clear what constraints to use and b) we usually want to fit a model to the result anyway so it is easier and clearer just to fit the model to the original observation.**

Forward-fitting

- The most common method of analyzing astronomical X-ray spectra is the "forward-fitting" method.
- This comprises the following steps...
  - Calculate a model spectrum
  - Multiply the result by the instrumental response matrix
  - Compare the result with the actual observed data by calculating some statistic
  - Modify the model spectrum and repeat till the best value of the statistic is obtained
- Usually the model spectrum is expressed in terms of a small number of parameters (eg power-law index and normalization for a simple power-law) and the "best-fit" values of these parameters and their confidence regions are obtained.
- The element (I, J) of the instrumental response matrix is proportional to the probability that a photon in energy range I will be assigned by the spectrometer to a channel J.



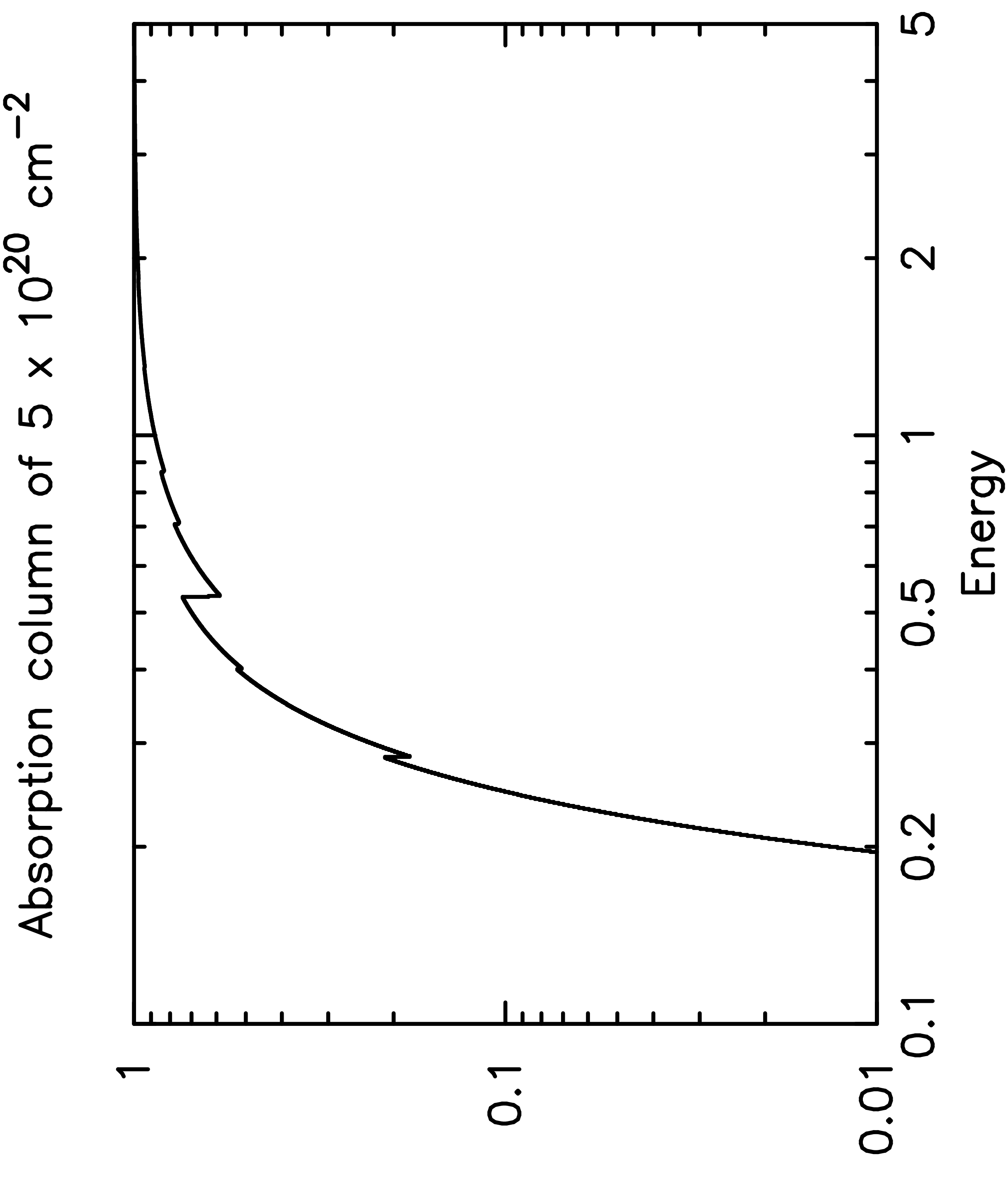
Define Model
Calculate Model
Convolve with detector response
Compare to data
Change model parameters

Models

"All models are wrong, but some are useful"
– George Box

- We build models up from components. The two basic types are additive (an emission component – e.g. bremsstrahlung, line, ...) and multiplicative (something that modifies the spectrum – usually a form of absorption.
- Basic emission models available are
  - blackbody
  - thermal bremsstrahlung
  - power-law
  - collisional plasma
  - lines

  There are a number of more specialised models covering topics such as accretion disks, comptonized plasmas, non-ionization equilibrium plasmas, multi-temperature collisional plasmas. There are more models in use than I know about because many people write their own.
- Most multiplicative models relate to photo-electric absorption due to material in the line-of-sight to the emitter. This may be local to the emitter or due to the Galactic ISM.


Absorption column of 5 x 10^{20} cm^{-2}
Transmission
1
0.1
0.01
Energy
0.1
0.2
0.5
1
2
5

Spectral fitting programs

- XSPEC – part of HEAsoft. General spectral fitting program with many models available.
- Sherpa – part of CIAO. Multi-dimensional fitting program which includes the XSPEC model library and can be used for spectral fitting.
- SPEX – from SRON in the Netherlands. Spectral fitting program specialising in collisional plasmas and high resolution spectroscopy.
- ISIS – from MIT Chandra HETG group. Mainly intended for the analysis of grating spectra.

Finding the best-fit

- Most methods used to find the best fit are local ie they use some information around the current parameters to decide where to try next. All these methods are liable to get stuck in local minima. Watch out for this ! Sometimes you can identify problems when you attempt to determine confidence regions and a new minimum is found.
- There are global minimization methods available – simulated annealing, genetic algorithm – but they require many function evaluations and there is still no absolute guarantee that they will find the true minimum.
- Remember - a minute of thought can beat hours of computation.

Max. likelihood in presence of background

- Wachter et al. (1979 ApJ 230, 274) is the best paper I know on fitting models to X-ray spectra subject from the Frequentist (non-Bayesian) perspective.

- It is almost universally ignored (or unknown).

- It presents a maximum likelihood statistic that includes background observations. It also argues that the usual modification of the χ^2 statistic in the presence of background is wrong.

- The basic trick is to use a model for the background with one parameter for each channel. These background parameters can be solved for analytically.

- I have recast Wachter et al. so it can be used in programs such as XSPEC and Sherpa. It is available in XSPEC (use stat wstat) and is coming soon to Sherpa. More details will be available on astro-ph when I have answered the referee's final comments.

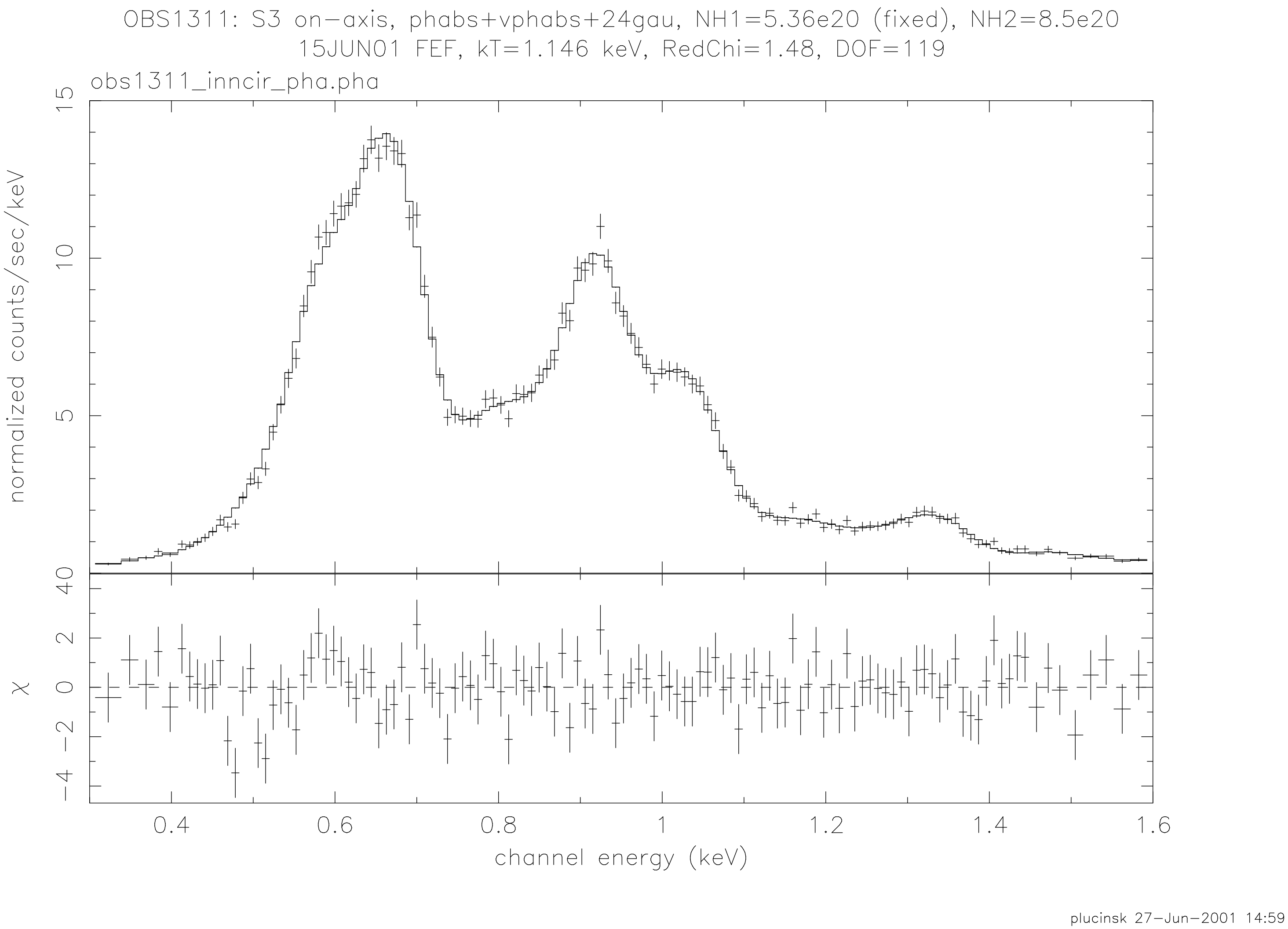


Figure 1: Fit to E0102 SNR spectrum at S3 aimpoint with Jun 15 PHA FEFs

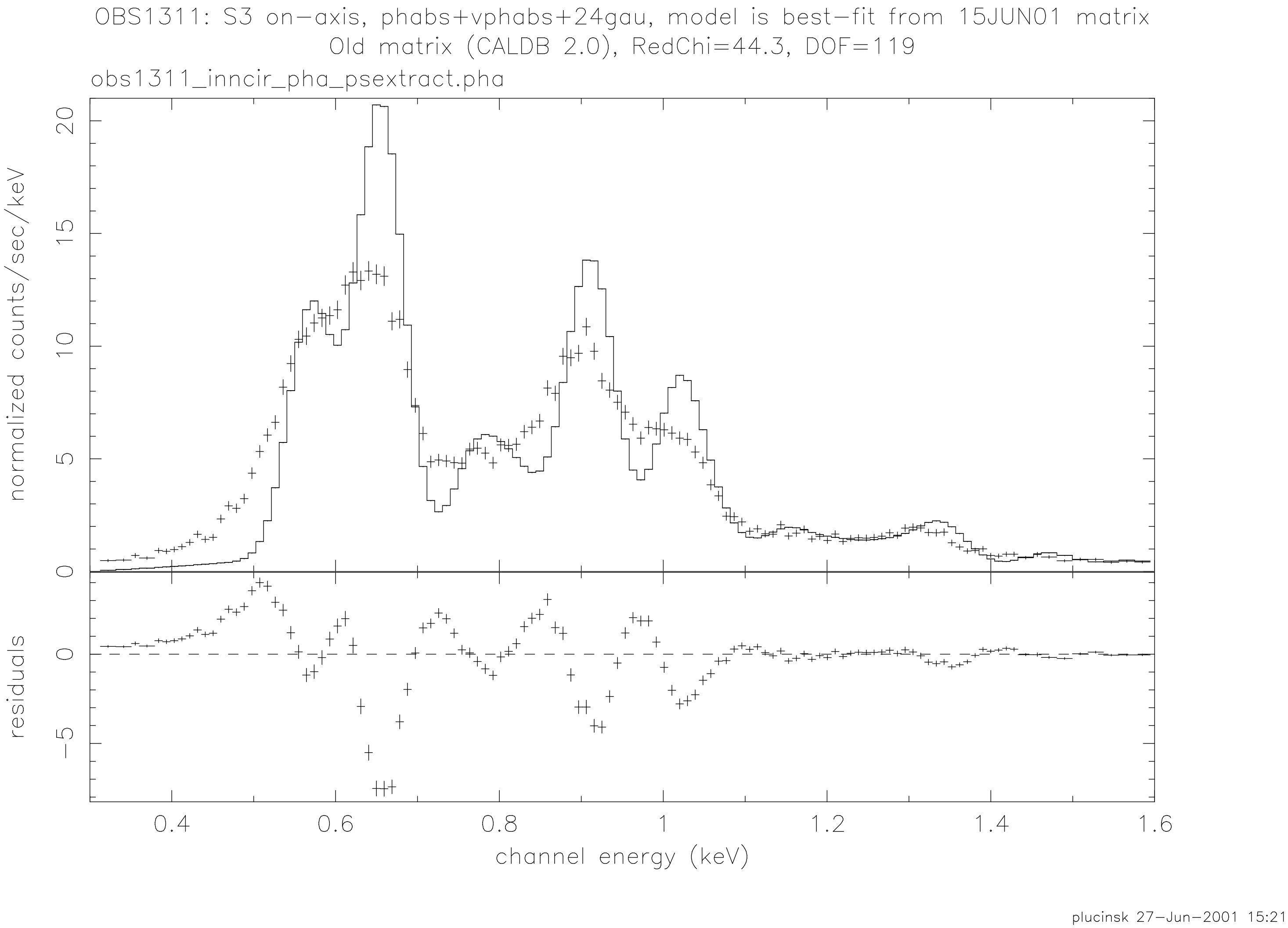


Figure 2: Same E0102 SNR model with old (CALDB 2.0) S3 FEFs

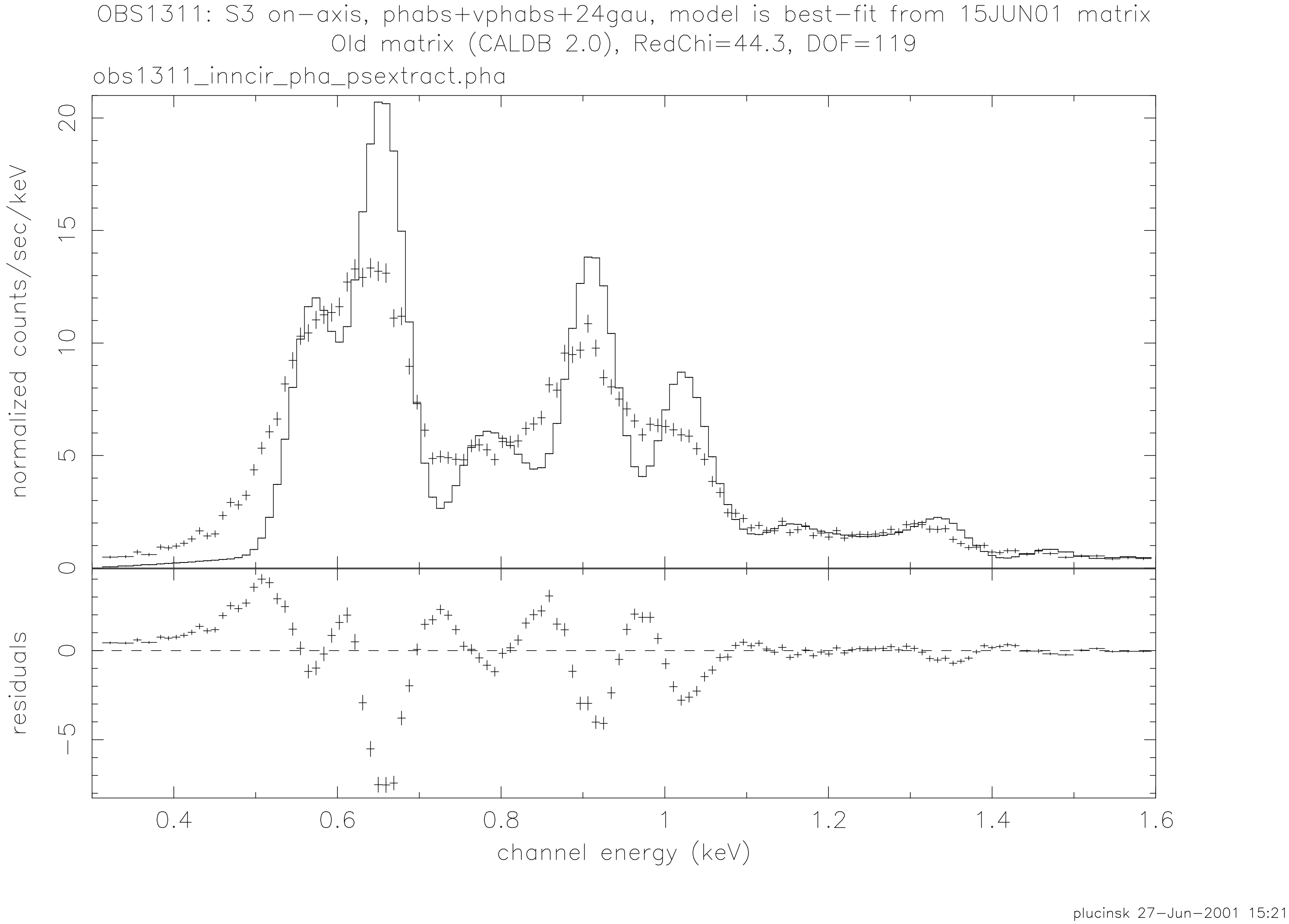


Figure 2: Same E0102 SNR model with old (CALDB 2.0) S3 FEFs