Continuum Radiation

Introduction:

1. Radiation Quantities
   - definitions
   - what can we learn from x-ray astronomy
   - continuum mechanisms
     - radiation by an electron
     - radiation by a population of electrons
     - what we learn
     - where this occurs
2. Bremsstrahlung
3. Compton Scattering
4. Synchrotron Radiation

Dan Schwartz
SAO/CXC
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Radiation Quantities

Allen, C.H., "The Atmospheres of the Sun and Stars" Ronald Press 1963, pp 141
Chandrasekhar, S., 1950 "Radiation Transfer" Dover, NY 1950

Everything we learn in X-ray Astronomy results from the emission and absorption of radiation.

We need to define certain radiation quantities. These include the quantities which we measure directly, and the idealized quantities which can be calculated from 1st principles physics.

The issues to keep in mind are

- Surface brightness (per unit solid angle) vs. a flux of parallel radiation?
- Distinguishing a quantity AT a specific narrow energy band vs a quantity integrated over energy.
- Reporting a quantity of photons vs a quantity of energy.

Radiation Quantities

The elementary quantity is the *SPECIFIC INTENSITY* $I_\nu(\Omega)$ or $I_\epsilon(\Omega)$,
the energy of radiation, of frequency ν or energy $\epsilon = h\nu$, which passes through a unit area, at polar angle θ, per unit solid angle per unit time.

$$dE_\nu = I_\nu \cos\theta \, dA \, d\nu \, d\Omega \, dt \tag{1}$$

We define the *SPECIFIC FLUX* or *FLUX DENSITY*

$$\mathcal{F}_\nu = \int_\Omega I_\nu \cos\theta \, d\Omega \tag{2}$$

In particular, we have the broadband, or integral, photon flux

$$F = \int_{\nu_1}^{\nu_2} \frac{\mathcal{F}_\nu}{h\nu} d\nu \tag{3}$$

This would be measured in a perfect detector with response from ν_1 to ν_2, and is the *total* flux if $\nu_1 \to 0$ and $\nu_2 \to \infty$.

What do we want to learn from continuum spectra?

1. Total power output
2. Energy budgets
3. Lifetimes
4. Temperatures (where applicable)
5. Magnetic field strengths (where applicable)

Continuum radiation comes virtually exclusively from electrons, since for a given force their acceleration is 1836 times higher than that of protons. We usually infer a power law distribution:

$$n(\gamma) = n_0\,\gamma^{-m} \tag{4}$$

electrons/cm^3 (unit γ), in a range γ_1 to γ_2. Since the total energy density in electrons is

$$U_e = \frac{n_0 mc^2}{m-2}(\gamma_1^{2-m} - \gamma_2^{2-m}), \tag{5}$$

we usually only care about either γ_1, if $m > 2$, or γ_2, if $m < 2$. (For m=2 we replace by a Log.) So we are learning about electrons, *AND* whatever exerts the force to make them radiate. In my exposition I'll start with the radiation by a single electron, then develop the volume emissivity of a population of electrons. I'll try to point out what we learn, and in what situations it may be encountered.

Electron Radiation

Radiation is emitted when electrons are accelerated. (cf. Jackson, 1962,"Classical Electrodynamics," ch 14; Lang, 1974, "Astrophysical Formulae," pp19.)

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c} \frac{|\mathbf{n} \times [(\mathbf{n} - \beta) \times \dot{\beta}]|^{\mathbf{2}}}{(1 - \mathbf{n} \cdot \beta)^5} \tag{6}$$

Expanding the motion as a Fourier integral gives the spectral intensity:

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2}{4\pi c} | \int_{-\infty}^{\infty} dt \frac{\mathbf{n} \times [(\mathbf{n} - \beta) \times \dot{\beta}]}{(1 - \mathbf{n} \cdot \beta)^2} e^{i\omega(t - (\mathbf{n} \cdot \mathbf{r})/c)} |^2 \tag{7}$$

In the non-relativistic case there is the classical Larmour formula:

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} \sin^2\theta \dot{v}^2 \tag{8}$$

KEY FEATURES:

- For extreme relativistic particles, $\beta \simeq 1$, the radiation is highly beamed within a cone of half angle $\theta \sim 1/\gamma$.
- Eq (7) $\rightarrow 0$ as $\omega \rightarrow \infty$, so total energy converges.

Continuum Mechanisms

References: Rybicki and Lightman, "Radiation Processes in Astrophysics,"
Tucker, "Radiation Processes in Astrophysics,"
Blumenthal and Gould, Rev. Mod. Phys.

We will consider the following processes:

1. Bremsstrahlung
   $\dot{\beta}$ due to collision with a proton (or heavy particle)
2. Compton Scattering
   $\dot{\beta}$ due to collision with a photon
3. Synchrotron Radiation
   $\dot{\beta}$ due to centripetal acceleration in a magnetic field

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Bremsstrahlung

In brems, the electron radiates a broad spectrum up to its total energy. In the X-ray range 0.1 to 10 keV, we will be interested in electrons up to a few 10's of keV, but still quite less than 511 keV. So we will take $\beta \ll 1$ in the general formula

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2}{4\pi c} \left| \int_{-\infty}^{\infty} dt \frac{\mathbf{n} \times [(\mathbf{n} - \beta) \times \dot{\beta}]}{(1 - \mathbf{n} \cdot \beta)^2} e^{i\omega(t - (\mathbf{n} \cdot \mathbf{r})/c)} \right|^2 \tag{9}$$

to give the *dipole approximation*

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2}{4\pi c} \left| \int_{-\infty}^{\infty} dt \, \mathbf{n} \times [(\mathbf{n} \times \dot{\beta}] \mathbf{e^{i\omega t}} \right|^{\mathbf{2}} \tag{10}$$

We can consider the collision taking place over a time $\tau = b/v$ for an impact parameter b, so that

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2}{4\pi c^3} \sin^2\theta \, |\Delta \mathbf{v}|^{\mathbf{2}} \quad \omega \ll 1/\tau$$

$$\frac{dI(\omega)}{d\Omega} = 0 \quad \omega \gg 1/\tau. \tag{11}$$

Bremsstrahlung Cross Section

Now consider an electron hitting a proton (avoids factors of Z^2). For any impact parameter b, a very quick encounter with a massive particle gives a deflection $|\Delta \mathbf{v}| = \frac{2e^2}{m\,v\,b}$. Integrating over angles gives the spectral power radiated in an encounter at b,

$$P(\nu, b) = \frac{8}{3\pi} r_0^2 \frac{e^2 c}{v^2 b^2}. \tag{12}$$

Integrate over all impact parameters to get the cross section for an electron to radiate at a frequency ν:

$$\sigma(\nu, v) = \int_0^\infty P(\nu, b) 2\pi b\, db = \frac{16}{3} e^2 c \frac{r_0^2}{v^2} \ln(b_{max}/b_{min}) \tag{13}$$

Where $b_{min} \simeq \hbar/mv$ from the uncertainty principle, and $b_{max} \sim v/\omega$. A quantum mechanical Born approximation, conserving energy, defines the log factor so that an electron of energy E radiates photons of energy ϵ proportional to $\ln \frac{(\sqrt{E}+\sqrt{E-\epsilon})^2}{\epsilon}$, for $\epsilon \leq E$.

KEY FACT: For a single electron, the ratio of bremsstrahlung to ionization $\simeq 10^{-3}(v/c)^2$.

Bremsstrahlung Applications

1. Solar x-rays
   - Thin target
   - Thick target
2. Ground x-ray calibrations
   - Electron impact used to excite lines, but extraction of continuum from data is essential for correct calibration of the response
3. Thermal Brems in clusters of galaxies
   - Total thermal energy
   - Abundances
   - Cooling lifetime
   - Pressure - interaction with radio structures
   - Total mass via hydrostatic equilibrium
   - Luminosity distance via S-Z effect

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Thermal Bremsstrahlung

Consider bremsstrahlung from a collection of electrons $N(v) = n_0\, f(v)$ with a normalized distribution f of velocities v. The spectral emissivity per unit volume is $s(\epsilon, v) = N(v)\, N_p v\sigma(\epsilon, v)$. For a distribution of electrons in statistical equilibrium at temperature T,

$$f(v) = 4\pi n_0 (\frac{m}{2\pi kT})^{3/2} e^{-mv^2/(2kT)} v^2, \qquad (14)$$

and if we take the electron and proton densities to be equal, the observed spectral energy intensity is

$$I_\Omega(\epsilon) = \int_0^L dx\, \int_{\sqrt{\frac{2\epsilon}{m}}}^\infty dv\, s(\epsilon, v) = \int_0^{L(\Omega)} dx\, \frac{10^{-11} n_0^2}{\sqrt{T}} e^{-\epsilon/kT}\, \overline{g(T,\epsilon)}$$

Where the averaged gaunt factor can be approximated by

$$g(T,\epsilon) = \frac{\sqrt{3}}{\pi} \ln \frac{4kT}{1.77\epsilon} \qquad \epsilon \ll kT$$

$$g(T,\epsilon) = (\epsilon/kT)^{-0.4} \qquad \epsilon \sim kT \qquad (15)$$

The total rate of energy emitted per unit volume is

$$P(T) = \int_0^\infty \frac{dI}{dL}\, d\epsilon = 1.4 \times 10^{-27} \sqrt{T} n_0^2\, \overline{g(T)} \qquad (16)$$

and the electron lifetime is

$$\tau = 3n_0\, kT/P(T) = 1.72 \times \sqrt{T}/n_0 \qquad (17)$$

Compton Scattering

We are interested in two applications.

Classical Compton Scattering: A photon loses energy in collisions with electrons. This modifies the photon spectrum (*Compton Reflection*) and transfers energy to the electrons (*Accretion disk coronae*).

Inverse Compton Scattering: Net energy is transfered from extreme relativistic electrons to photons. The produces observed X-rays in radio source cores, jets, and lobes; it may be significant in pulsars and cores of all AGN.

Compton interaction

A low energy photon of incident energy $x'\,mc^2$ will Compton scatter through an angle θ with cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\,r_0^2\,(1+\cos^2\theta). \tag{18}$$

The total cross section is the Thomson cross section $\sigma_T = \frac{8\pi}{3}\,r_0^2 = 6.65\times10^{-25}\mathrm{cm}^2$.
Kinematics gives the energy of the scattered photon as $x\,mc^2$, where

$$x = \frac{x'}{1+x'(1-\cos\theta)} \tag{19}$$

KEY FEATURES:

- $x'/(1+2x') \le x \le x'$
- for $\theta \approx 0, \quad x \simeq x'$
- for $x' \ll 1, \;\; x \simeq x'$
- for $x' \gg 1$, and θ not near 0, $x \simeq 1$

For completeness, the Klein-Nishina cross section valid for arbitrary energies is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\,r_0^2\,(\frac{x}{x'})^2(\frac{x'}{x}+\frac{x}{x'}-\sin^2\theta) \tag{20}$$

Accretion Disk Coronae

One photon in one scattering transfers an average energy

$$\int_\Omega (x' - x)\frac{1+\cos^2\theta}{2}\,\sin\theta\,d\theta\,d\phi \;\simeq (x')^2 \qquad (21)$$

So a blackbody distribution of photons transfers energy to cold electrons at a rate

$$4\frac{\sigma_T\,n_0}{mc}kT'\rho, \quad \rho = aT^4$$

and in general $T' = T_{photons} - T_{electrons}$. This can provide energy from a population of photons to heat electrons. Typically, an accretion disk corona is much hotter than an optically thick disk which may underlie it, so that net energy flows from the electron population and modifies the X-ray spectrum.

This complex process is governed by the Kompaneets diffusion equation, (cf. work by Titarchuk; Sunyaev; and collaborators).

To fair accuracy, low energy photons form a power spectrum with an energy index

$$\alpha = \sqrt{9/4 \;+ 1/y} - 3/2 \;\text{ if } kT \leq mc^2 \text{ and } y = \mathcal{O}(1)$$

$$\text{where } y = \frac{kT_e}{mc^2}\frac{3\tau^2}{\pi^2}$$

For $y \gg 1$, the spectrum has a Wein exponential cutoff.

Inverse Compton Scattering

Reference: Felten and Morrison, 1966, ApJ, 146, 686.

When the electrons are highly relativistic, collision with a low energy photon can result in the latter being scattered into the X-ray band. The formalism follows fairly directly from classical Compton scattering, by transforming to the rest frame of the electron before the collision, and back to the observer's frame afterwards.

In the electron rest frame the incident photon has energy $x_e = \gamma\, x_o\,(1 - \beta\cos\theta)$. If the photon to electron velocity angle is θ_o in the lab, relativistic aberration gives
$\tan\theta_e = \frac{\sin\theta_o}{\gamma(\cos\theta_o + \beta)}$, which is very close to 0 unless $\theta_o \approx \pi$.

After the collision, the photon has scattered through an angle ϕ' to an energy $x'_e = \frac{x_e}{1 + x_e(1 - \cos\phi')}$ and in the observer's frame $x'_o = \gamma\, x'_e(1 + \beta\cos\theta'_e)$.

KEY RESULT: Taking all the angles to be near 0, so all the cos are near 1, we have
$x'_o \simeq \gamma^2\, x_o$.

Inverse Compton: Power and Spectrum

Reference: Blumenthal and Gould, 1970, Rev Mod Phys

We see that electrons with $\gamma \sim 10^2$ can scatter infrared photons to the X-ray band, and with $\gamma \sim 10^3$ would scatter the 3° microwave background photons to X-ray energies.

The rate of scattering is $\sigma_T \, v \, n_0 = \sigma_T \, c \, \frac{\rho}{x_o}$,

so the total power is $P_C(\gamma) = \frac{4}{3} \gamma^2 \, \sigma_T \, c\rho = 2.66 \times 10^{-14} \, \gamma^2 \, \rho$

and the electron lifetime is $\tau_C = \gamma \, mc^2 / P_C(\gamma) = \frac{2 \times 10^{-19} \mathrm{sec}}{\gamma \, \rho [\mathrm{eV/cm}^3]}$

The actual spectrum radiated by an electron is not monochromatic. Blumenthal and Gould give the shape of the photon spectrum radiated by an electron of energy γ hitting a photon of energy x as a parameterized function of the variable
$y = \frac{x'}{4x\gamma^2}$:

$$f(y) = 2y \ln y + y + 1 - 2y^2 \tag{22}$$

Inverse Compton from a Distribution of Electrons

In astronomical applications, inverse Compton is complicated by the need to integrate over distributions of electrons and target photons, (as well as scattering angles). Although the spectrum of a single electron is broad, as we just saw, when we integrate over a power law population of electrons $n(\gamma) = n_0\, \gamma^{-m}$, we often use the approximation (Hoyle, 1960) that the electron spectrum is a delta function at some mean energy. Following Felten and Morrison (1966) we take $E_C = \frac{4}{3}\gamma^2\, x\, mc^2$.

If we average over a blackbody spectrum of target photons, their mean energy is $x\, mc^2 = 2.7kT$. Taking a path length L through the source (as a short cut for integrating), we have the specific energy intensity

$$I(E_C) = L \int_1^\infty d\gamma\, P_C(\gamma)\, n(\gamma)\, \delta(E_C - 3.6\gamma^2\, kT)$$
$$= 1000(56.9)^{3-m} n_0 L \rho\, T^{\frac{m-3}{2}}\, E^{\frac{1-m}{2}}\ \mathrm{eV/(eV\ \ cm^2\, sec\, ster)}$$

KEY FEATURES:

- For an electron spectral index m, the X-ray energy index is $\alpha = (m-1)/2$.
- If we can estimate ρ, $n_0\, L$ is the only unknown.

Synchrotron Radiation

Synchrotron is the ubiquitous radio emission mechanism. It provides unique information on the magnetic field strengths, and on field directions via the observation of polarization. It can be an important X-ray emission mechanism in pulsars and supernovae remnants, and for X-rays from the jets, hot-spots, lobes, and perhaps cores of extra-galactic radio sources. There is a powerful synergy when we can observe Compton X-ray emission from the same population of electrons which give radio synchrotron emission. I will quote some formulas and point out the analogies to inverse Compton emission. The total power radiated by an electron is

$P_S(\gamma) = \frac{2}{3} r_0^2\, c\gamma^2 H_\perp = 9.89 \times 10^{-16}\gamma^2\, H_\perp[\mu\text{G}]$ eV/sec.

KEY FEATURES:

- The relative powers $\frac{P_S}{P_C} = \frac{H^2/8\pi}{\rho}$ are the ratio of energy densities in magnetic field and in photons.
- Electron lifetime is $\tau_S = \frac{8\times10^{20}}{\gamma H_{\mu G}^2}$ seconds.

Synchrotron Spectra

The exact spectrum radiated by an electron is

$$F_S(\nu) = \frac{\sqrt{3}\, e^3}{mc^2} H_\perp \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta \qquad (23)$$

where ν_c is the characteristic emission frequency $\nu_c = 4.2\gamma^2 H_\perp[\mu G]$.

Although $F_S(\nu)$ is a maximum at $.3\nu_c$, to integrate over a power law spectrum of electrons we approximate the emission of an electron as a delta function at ν_c, and calculate the specific energy intensity:

$$\begin{aligned} I_S(\nu) &= L \int_0^\infty d\gamma\, n(\gamma)\, P_S(\gamma)\delta(\nu - \nu_c) \\ &= 4.8 \times 10^6 (490)^{3-m}\, n_0\, L H_{\mu G}^{(1+m)/2} \nu_{MHz}^{(1-m)/2} \qquad (24) \end{aligned}$$

KEY FEATURES:

- Spectral dependence is the same as for inverse Compton
- H is the only new variable. If we can measure IC and synchrotron radiation, we can solve for all the intrinsic parameters, i.e. H and n_0

Related Topics

We were not able to cover

1. **Black Body Radiation**. Occurs in neutron stars, white dwarfs, locally in accretion disks, and the cosmic microwave background. The specific energy intensity is the Planck function:
   $B_T(\nu) = \frac{2h\nu^3}{c}(\exp(\frac{h\nu}{kT}) - 1)^{-1}$ ergs/(Hz cm^2 sec ster).
2. **Equilibrium Distribution of Electrons**
   $\frac{\partial N(E,t)}{\partial t} + \frac{\partial}{\partial E}\dot{E}\, N(E,t) = S(E,t)$
3. **Relativistic Beaming** Use the effective Doppler factor
   $\delta = \frac{1}{\Gamma}(1 - \beta\cos\theta)^{-1}$, and we observe $I_o = I_e \delta^{3+\alpha}$
4. **Absorption**
   - Atomic absorption by cold or warm gas
   - Synchrotron self-absorption in radio sources
5. **Radiation Transfer**

$$\frac{\partial\ I(\tau,\mu,x)}{\partial s} = -\sigma(x)\rho\, I(\tau,\mu,x) + \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}\frac{d\sigma(x')}{d\Omega} I(\tau,\mu',x')d\mu' d\phi \qquad (25)$$

References:

1. Allen, C.H., "The Atmospheres of the Sun and Stars" Ronald Press 1963, pp 141
2. Blumenthal and Gould, 1970 Rev. Mod. Phys., 42, p 236.
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8. Rybicki and Lightman, "Radiation Processes in Astrophysics," Wiley-Interscience, 1979.
9. Sunyaev, R.A., Titarchuk, L. G., 1980, A&A, 86, 121; 1985, A&A, 143, 374.
10. Tucker, "Radiation Processes in Astrophysics," MIT Press, 1975.