Let's assume there are source spectral files which you want to combine,

$$S_1\_spec, S_2\_spec, , , \text{and } S_n\_spec,$$

and these files have the EXPOSURE values

$$X_1, X_2, , \text{ and } X_n,$$

and BACKSCAL values

$$Y_1, Y_2, , \text{ and } Y_n,$$

respectively. These spectral files should have the unit COUNTS (not COUNTS/sec), as expected being produced with standard HEASARC software packages.

Corresponding background files are

$$B_1\_spec, B_2\_spec, , \text{ and } B_n\_spec,$$

which have the EXPOSURE values

$$X_1', X_2', , \text{ and } X_n',$$

and BACKSCAL values

$$Y_1', Y_2', , \text{ and } Y_n',$$

respectively.

For each pair of the source and background spectral files, XSPEC calculates the following quantities to be fitted with models.

$$\begin{aligned} &\frac{S_1\_spec}{X_1} - \frac{Y_1}{Y_1'}\frac{B_1\_spec}{X_1'}, \\ &\frac{S_2\_spec}{X_2} - \frac{Y_2}{Y_2'}\frac{B_2\_spec}{X_2'}, \\ &\qquad\qquad\qquad\qquad \ldots, \\ &\frac{S_n\_spec}{X_n} - \frac{Y_n}{Y_n'}\frac{B_n\_spec}{X_n'}. \end{aligned}$$

From these quantities, one can make a single quantity,

$$\frac{S_1\_spec + S_2\_spec, , +S_n\_spec}{X_1 + X_2+, , +X_n} -$$
$$\frac{1}{X_1 + X_2+, , +X_n}\left\{\frac{X_1}{X_1'}\frac{Y_1}{Y_1'}B_1\_spec + \frac{X_2}{X_2'}\frac{Y_2}{Y_2'}B_2\_spec+, , +\frac{X_n}{X_n'}\frac{Y_n}{Y_n'}B_n\_spec\right\}. \tag{1}$$

This is the quantity we should make to be fitted with models.

With **mathpha**, $S_1\_spec, S_2\_spec, ,$, and S_n_spec can be summed with the option "exposure=CALC" and "backscal=NONE" (or "backscal=1"), so that the combined spectrum has the EXPOSURE value $X_1 + X_2+, , +X_n$ and the BACKSCAL value unity.

Let's put the weights for the background spectra to combine with $C_1, C_2, , , C_n$ and the BACKSCAL value for the combined background spectrum $(BACKSCAL)_{bgd}$.

With the option "exposure=CALC", the EXPOSURE of the combined background spectrum will be

$$C_1X_1' + C_2X_2'+,,+C_nX_n'.$$

By properly choosing $C_1, C_2,,, C_n$, as shown below, this can be made equal to the desirable EXPOSURE value,

$$X_1' + X_2'+,,+X_n'.$$

So we have,

$$C_1X_1' + C_2X_2'+,,+C_nX_n' = X_1' + X_2'+,,+X_n'. \tag{2}$$

Consequently if the combined background spectrum thus made is subtracted from the combined source spectrum, XSPEC calculates the quantity,

$$\frac{S_1\_spec + S_2\_spec,,+S_n\_spec}{X_1 + X_2+,,+X_n} -$$

$$\frac{1}{X_1' + X_2'+,,+X_n'} \frac{1}{(BACKSCAL)_{bgd}} (C_1B_1\_spec + C_2B_2\_spec+,,+C_nB_n\_spec). \tag{3}$$

The values (1) and (3) should agree. Hence we have,

$$\begin{aligned}
C_1 &= (BACKSCAL)_{bgd} \left(\frac{X_1' + X_2'+,,X_n'}{X_1 + X_2+,,X_n}\right) \left(\frac{X_1}{X_1'}\right) \left(\frac{Y_1}{Y_1'}\right), \\
C_2 &= (BACKSCAL)_{bgd} \left(\frac{X_1' + X_2'+,,X_n'}{X_1 + X_2+,,X_n}\right) \left(\frac{X_2}{X_2'}\right) \left(\frac{Y_2}{Y_2'}\right), \\
& \qquad \ldots, \\
C_n &= (BACKSCAL)_{bgd} \left(\frac{X_1' + X_2'+,,X_n'}{X_1 + X_2+,,X_n}\right) \left(\frac{X_n}{X_n'}\right) \left(\frac{Y_n}{Y_n'}\right)
\end{aligned} \tag{4}$$

Equation (2) and equations (4) can be solved for $(BACKSCAL)_{bgd}$ and $C_1, C_2,, C_n$. Then finally, we get,

$$\begin{aligned}
(BACKSCAL)_{bgd} &= \frac{X_1 + X_2+,,+X_n}{X_1(Y_1/Y_1') + X_2(Y_2/Y_2')+,,+X_n(Y_n/Y_n')} \\
C_1 &= \frac{X_1' + X_2'+,,+X_n'}{X_1(Y_1/Y_1') + X_2(Y_2/Y_2')+,,+X_n(Y_n/Y_n')} \left(\frac{X_1}{X_1'}\right) \left(\frac{Y_1}{Y_1'}\right) \\
C_2 &= \frac{X_1' + X_2'+,,+X_n'}{X_1(Y_1/Y_1') + X_2(Y_2/Y_2')+,,+X_n(Y_n/Y_n')} \left(\frac{X_2}{X_2'}\right) \left(\frac{Y_2}{Y_2'}\right) \\
& \qquad ,,, \\
C_n &= \frac{X_1' + X_2'+,,+X_n'}{X_1(Y_1/Y_1') + X_2(Y_2/Y_2')+,,+X_n(Y_n/Y_n')} \left(\frac{X_n}{X_n'}\right) \left(\frac{Y_n}{Y_n'}\right).
\end{aligned}$$

These are the values you need to ensure the correct amount of background subtraction.