X-ray Timing in Astrophysics

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Astronomy is an *observational*, not *experimental* science.

Mostly done by characterizing the electromagnetic field impinging on Earth with a few exceptions (cosmic rays, neutrinos, gravitational waves).

The EM field can be characterized by intensity a function of: angle, energy (i.e. frequency), polarization, and time.

Here, we will focus on the time domain, in other words, source variability.
In the X-ray band, detectors are sensitive to individual photons, which each carry significant energy \( (E = h\nu) \).

- \( 1 \text{ keV} = 1.6 \times 10^{-9} \text{ erg} = 2.24 \times 10^{17} \text{ Hz} = 1.24 \times 10^{-7} \text{ cm} \)

Detectors can record the arrival time, energy, and direction of each photon (and perhaps polarization in the future).

2 seconds of raw data from GRS1915+105
Aside on Photon Statistics

Warning: Because we are counting individual photons, the relevant statistics are Poisson, not Gaussian.
Source variability probes geometry of the emitting region in a way spectra cannot.

Fastest time scales probe the smallest time size scales.

- Accretion dynamics near event horizon of BH or surface of NS, burning fronts propagating around NS, magnetic reconnection bursts on a magnetar.

Coherent pulsations allow extremely precise measurements.

- Orbital period and evolution, accretion torques, rotational glitches.
Rotational Periods:

ms - s for NS/WD
hr - days for Stars

Accretion Time Scales:

Dynamical, Thermal, & Viscous Time Scales (e.g. QPOs, outburst timescales)
ms – days for NS/BHC
minutes – years for AGN

Orbital Time Scales:

minutes to days for NS/BHC
Suber-orbital periods:
weeks – months

X-ray Bursts & Superbursts
Characteristic Time Scales

\[ \tau \geq \frac{R}{v}, \quad v \leq c, \quad R \geq \frac{2GM}{c^2} \]

- AGN \((10^8 \, M_\odot) \Rightarrow \tau > 1000 \, \text{s}\)
- Black Hole \((10 \, M_\odot) \Rightarrow \tau > 100 \, \mu\text{s}\)
- Neutron Star \((1.4 \, M_\odot) \Rightarrow \tau > 15 \, \mu\text{s}\)

These are the fastest achievable time scales. In reality, there is variability on a range of time scales.
Software Tools

- **HEASoft (FTOOLS)**
  - Distributed by NASA’s HEASARC
    - [http://heasarc.gsfc.nasa.gov/docs/software/lheasoft/](http://heasarc.gsfc.nasa.gov/docs/software/lheasoft/)
  - Supports many mission formats (RXTE, Swift, etc...) and generic FITS files

- **SITAR**<http://space.mit.edu/CXC/analysis/SITAR>
  - Being developed by Mike Nowak

- Or, “roll your own” as many people do
  - Custom C or FORTRAN code
  - IDL or MATLAB
  - Python + SciPy&Matplotlib
Simplest Tool: A Lightcurve

- Select photons from an energy range of interest and “bin” them into evenly spaced time bins with $N_i$ counts/bin

- *Tip:* Always choose integer multiple of “natural” time unit for binning

- Don’t bin more than you have to – save it for subsequent analysis

- Be careful to normalize by exposure time

- Once you convert from counts/bin to rates or subtract any background or DC component, error is no longer $\sqrt{N_i}$
Length & Binning Determine Limits

☐ Lowest Frequency: \( f_{\text{long}} = 1/T \)

☐ Highest Frequency: Nyquist Frequency, \( f_{\text{Nyq}} = 1/(2\Delta t) \)

☐ Basic Question, is the variance: \( \sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 \) greater than expected from Poisson noise?

☐ \( \sigma \) = Root Mean Square Variability
Fourier Transform Methods

- The workhorse of the timing world
- Describes how variability power is distributed as a function of frequency
Fourier Transform Definition

A Fourier Transform decomposes a time series into “sine waves” of different frequencies

Power Density Spectrum (PDS) is the squared Fourier amplitude, properly normalized

Lightcurve with $N$ bins, comprised of counts, $x_i$, becomes power spectrum, with $N/2+1$ independent amplitudes

Discarding phases throws out information $\Rightarrow$ power spectra are not unique!

Know Your Normalization!!! Various FFT Routines Have Different Ones! (FTOOLS routine `powspec` gives you a choice)

- “One-sided” Leahy (mean power = 2): $P_j = 2|X_j|^2 / N_{ph}$
- “One-sided” (RMS/mean)$^2$/Hz: $P_j = 2|X_j|^2 / (N_{ph} \times <\text{Rate}>)$
Useful Theorems

- **Fourier Transform is a linear transform**
  \[ a x(t) \Leftrightarrow a X(f) \]

- **Real-valued data:**
  \[ x_k \in \mathbb{R} \Rightarrow X_{N-j} = X_j^* \text{ where } j \in [1, N/2 - 1] \]

- **Parseval’s Theorem**
  \[ \sum_{k=0}^{N-1} |x_k|^2 = \frac{1}{N} \sum_{j=0}^{N-1} |X_j|^2 \]

- **Shift**
  \[ x(t - t_0) \Leftrightarrow X(f)e^{2\pi if t_0} \]
FAST Fourier Transforms

- FFT algorithm (Cooley & Tukey 1965) transformed problem from $O[N^2]$ to $O[N \log_2(N)]$ which greatly increased the usefulness of Fourier techniques.

- Current state-of-the-art is the FFTW ("Fastest Fourier Transform in the West") library by Frigo & Johnson (MIT).
  - Many FFTs require or strongly prefer $N=2^n$, but FFTW works well with any small prime factors and still works even with $N=prime$.
  - It is highly portable (Linux/Mac/Windows/...) and is close to the fastest possible FFT on every platform with no special effort.

- Get it! <http://www.fftw.org>
Coherent Signals

- Much analysis involves “coherent” signals, i.e. periodic signals whose phase is constant over the relevant duration
  - Or, equivalently, where a time transformation (sometimes called a “timing model”) can be determined that makes the signal coherent

- Examples:
  - Pulses from rotating pulsars
  - Orbital modulation or eclipses
  - Precession periods
Epoch Folding

○ Bin photons according to phase with respect to a *known* period $P$ (or a more complicated timing model)

○ Significance of variability at that period can be assessed by doing a $\chi^2$ test against a null hypothesis of constant rate.
Epoch Folding Searches

- Perform epoch folding at a large number of trial periods, and look for trials with large $\chi^2$
- Good for non-sinusoidal variations, and when there are data gaps or complicated window functions
- Can be slow to explore a large range of periods
  - Requires $N_{ph} \times N_{per}$ operations: $\text{fmod}(t_{ph}, P)$
**FFT Searches**

**Pros**
- MUCH faster than epoch folding searches in many cases
- Searches all possible frequencies simultaneously

**Cons**
- Potentially large memory requirements
- Requires harmonic summing for non-sinusoidal signals
Statistics of Power Spectra

How do you determine the significance of peaks found in power spectra?

- Distribution of $P_k$ is $\chi^2/MW$ with $2MW$ D.O.F., where $MW$ is the number of power spectra summed.

So, just compute the probability of a false occurrence: $\Pr(P_k > \text{thresh})$

- Number of trials is critical!
- Distribution has a LONG tail!

PDF $[\chi_r^2(x)] = \frac{2^{-r/2}e^{-x/2}x^{r/2-1}}{\Gamma(r/2)}$
Decoherence

FFT and simple epoch-folding searches require the signal to be coherent throughout the interval being considered.

But, this might not be the case because of:

- Orbital Doppler shifts from a binary system
- Intrinsic period derivative of the source
- Satellite or Earth motion that isn’t fully compensated for

Searching still possible with several techniques.
Acceleration Searches

- Attempt to transform the time series into a frame where the signal is coherent
- Stretch the time series according to a set of trial accelerations, or matched filter in the Fourier domain
  - Assumes constant acceleration during observation
- Only works when higher order terms can be ignored (e.g. when $T_{\text{obs}} < P_{\text{orb}}/10$)
When $T_{\text{obs}} > P_{\text{orb}}$, the response to the FFT of a sinusoidal signal is analytically calculable as a Bessel function.


- Perform matched filter in the Fourier domain

- Recovers substantial fraction of fully coherent search sensitivity at a tiny fraction of the computational cost!
Coherent timing over long time baselines is very powerful and precise since every cycle is accounted for.

Goal: To determine a timing model that accounts for all of the observed pulse arrival times (TOAs).

Parameters that can be determined:

- Spin ($\nu$, $\nu'$, ... $\Rightarrow$ torques, magnetic fields, ages)
- Orbital ($P_{\text{orb}}$, $T_0$, $e$, $\omega$, $a_x \sin i$, GR terms)
- Positional ($\alpha$, $\delta$, $\pi$, proper motion)
Measuring a TOA

- Measure phase shift between measured TOA and a template profile

- Application of the FFT shift theorem (and linearity)

\[ x(t - t_0) \Leftrightarrow X(f) e^{2\pi i ft_0} \]

- TOA = \( T_{\text{obs}} + \Delta t \)
Arrival times at Earth or spacecraft must be converted to a nearly inertial frame before attempting to fit a simple timing model.

Remove effects of observer velocity and relativistic clock effects.

Convenient frame is the Solar System Barycenter.
Fitting TOAs to a Timing Model

\[ \phi(t) = \phi(0) + \nu t + \frac{1}{2} \dot{\nu} t^2 + \frac{1}{6} \ddot{\nu} t^3 + \ldots \]

Full model can include spin, astrometric, binary, and other parameters.

Goal: Find parameter values that minimize the residuals between the data and the model.
Tools for Fitting Timing Models

○ **Tempo** <http://pulsar.princeton.edu/tempo/>
  - Developed by Princeton and ATNF over 30+ years
  - Well tested and heavily used
  - Based on TDB time system
  - **But, nearly undocumented, archaic FORTRAN code**

○ **Tempo2** <http://www.atnf.csiro.au/research/pulsar/tempo2/>  
  - Developed at ATNF recently (still in beta test)
  - Based on TCB time system (coordinate time based on SI second)
  - Well documented, modern C code, uses `long double` (128 bit) throughout
  - Easy plug-in architecture to extend capabilities
  - **But, not well tested, still in development**

**Time Systems**

- TAI = Atomic time based on the SI second
- UT1 = Time based on rotation of the Earth
- UTC = TAI + "leap seconds" to stay close to UT1
- TT = TAI + 32.184 s
- TDB = TT + periodic terms to be uniform at SSB
- TCB = Coordinate time at SSB, based on SI second
Aperiodic Variability

- The broadband power spectrum can characterize:
  - Total (excess) variability
  - Power spectral slopes and breaks (special time scales)
  - Quasiperiodic oscillations (QPOs)
    - Random walks in phase or frequency
    - Finite lifetime of processes
    - Amplitude modulation

“Quality factor” $Q = f_0/\Delta f$
Rebinning and Averaging

- Single FFT bin is a terrible estimator of the PSD, because of the huge variance
- Making FFT longer doesn’t help; just samples frequencies more finely
- Solutions:
  - Average adjacent frequency bins (often done logarithmically)
  - Average PSDs of multiple data segments

“Twin” kHz QPOs

“Band-Limited Noise”
After subtracting Poisson level, you can fit models

\[ P'_j = (P_j - P_{\text{noise}}) \pm P_j / \sqrt{N_{\text{avg}}} \]

Popular choice currently is a sum of Lorentzians


\[ L(x) = \frac{\Gamma}{2\pi \left( (x - x_0)^2 + \frac{\Gamma^2}{4} \right)} \]
Dead Time Effects

- Detector “Deadtime” is when the detector can’t detect events, either:
  - For a period of time after an event
    - Paralyzable (event during deadtime extends deadtime)
    - Non-paralyzable (events during deadtime have no effect)
  - Or, for a detector reason, such as readout intervals

- Deadtime modifies the power spectrum of Poisson noise from the expected $P_{Leahy} = 2$ (usually to something $< 2$)

Advanced Topic: Unevenly Sampled Data

- Lomb Periodogram
- Bayesian Methods
- Wavelets
Review/Tips

- Coherent pulsation (e.g. pulsar) best done with no rebinning
- Pulsar timing is a powerful and precise tool
- QPO searches need to be done with multiple rebinning scales
- Beware of spurious signals introduced by:
  - Instrument (read times, clock periods, ...)
  - Dead time
  - Spacecraft orbit (background rate variations)
  - Diurnal/Annual effects
Proposal Estimates

- Detecting broad band noise (or QPO) at the \( n_\sigma \) confidence level

- For broad band timing, you win more with rate than time

\[
\text{RMS}_{\text{limit}}^2 \approx 2n_\sigma \sqrt{\Delta f} / \sqrt{\text{Rate}^2 \times T_{\text{total}}}
\]

- Detecting coherent pulsations

\[
f_p^{\text{limit}} = 4n_\sigma / (\text{Rate} \times \text{Time})
\]
  ○ Superb overview of spectral techniques!

Press et al., “*Numerical Recipes*”
  ○ Clear, brief discussions of many numerical topics

  ○ FFT & PSD Statistics

  ○ Epoch Folding

  ○ Epoch Folding Statistics

  ○ Noise Statistics

  ○ Timing tutorial + coherence techniques
Data Exercises

○ Get a computer with HEASoft installed
  ○ Linux/Mac/Sun/OSF etc... (Windows only under Cygwin)

○ Measure the pulsations from Sco X-1

○ Find the 0.1 Hz QPO in XTE J1118+480