X-ray Timing Analysis

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The Questions That We'd Like to Answer:

- Does My Source Vary?
- On What Time Scales Does it Vary?
- Are the Variations Periodic or Aperiodic?
- How Do Different Energy Bands Relate to One Another?

*(With Some Judicious Stealing of Slides from Z. Arzoumanian's 2003 X-ray Astronomy School Talk)

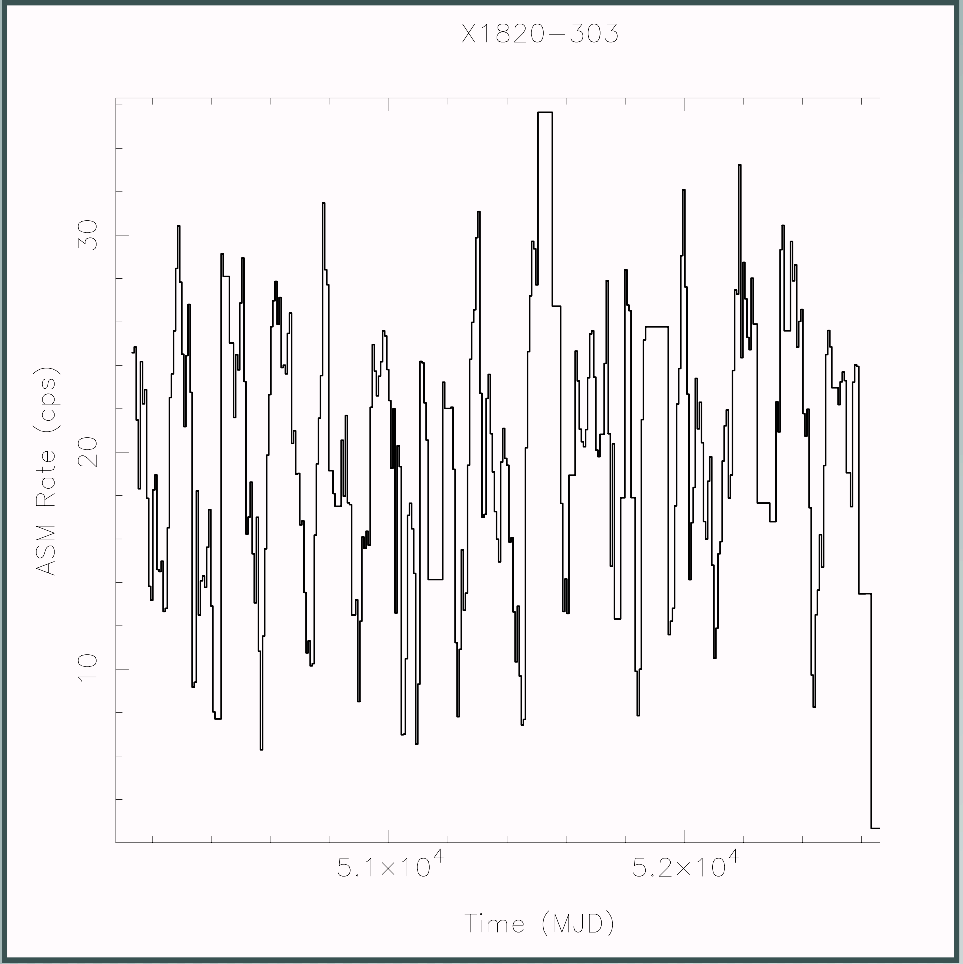
Characteristic Time Scales:

$\tau \geq R/V, \quad V \leq c, \quad R \geq 2\,GM/c^2$

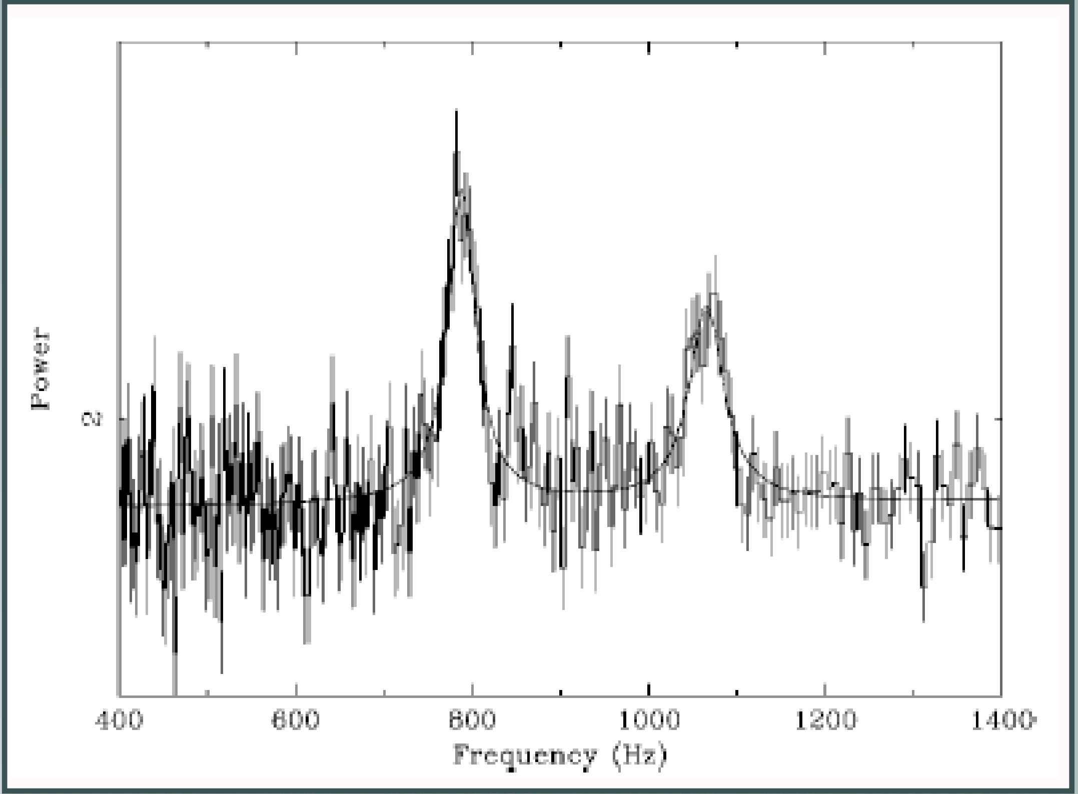
$\tau \geq$ 1000 sec	10^8 $M_\odot$ (AGN)
$\tau \geq$ 100 μsec	10 $M_\odot$ (BHC)
$\tau \geq$ 75 μsec	1.4 $M_\odot$ (NS)

These are the Fastest Achievable Time Scales. In Reality, There Can be Variability on a Range of Time Scales.

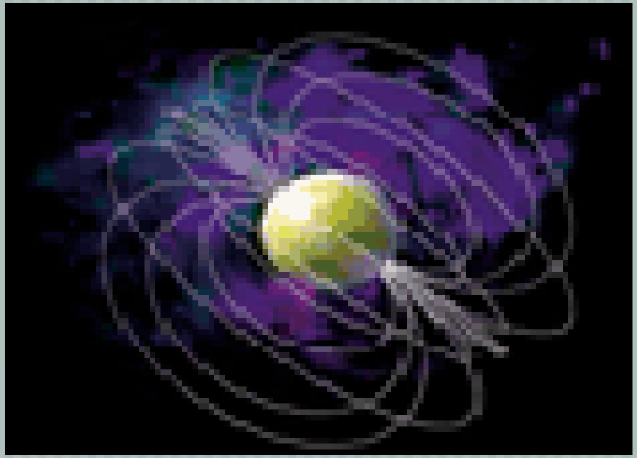
X 1820-303 (11 minute orbit)



180 Day Superorbital Period



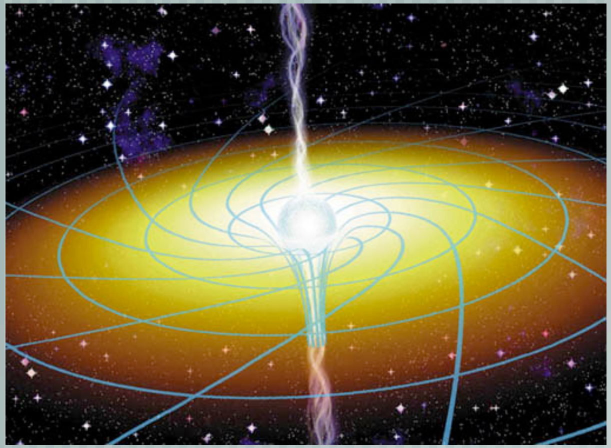
msec QPO



Rotational Periods:

msec - sec for NS/WD

hr - days for Stars



Accretion Time Scales:

Dynamical, Thermal, Viscous
Time Scales

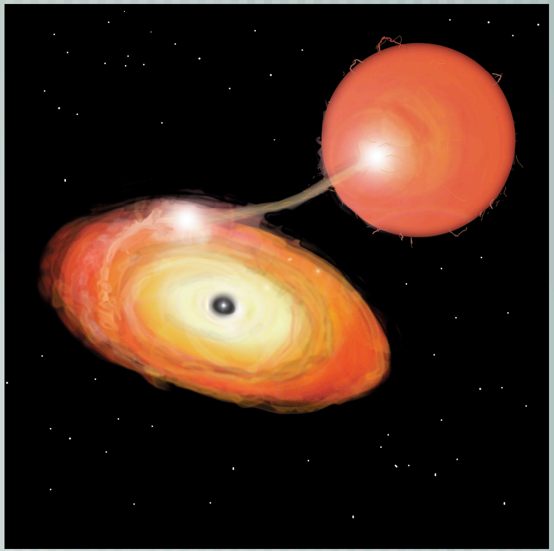
msec - days for NS/BHC

minutes - years for AGN

Orbital Time Scales:

minutes to days for NS/BHC

Suber-orbital periods:
weeks to months

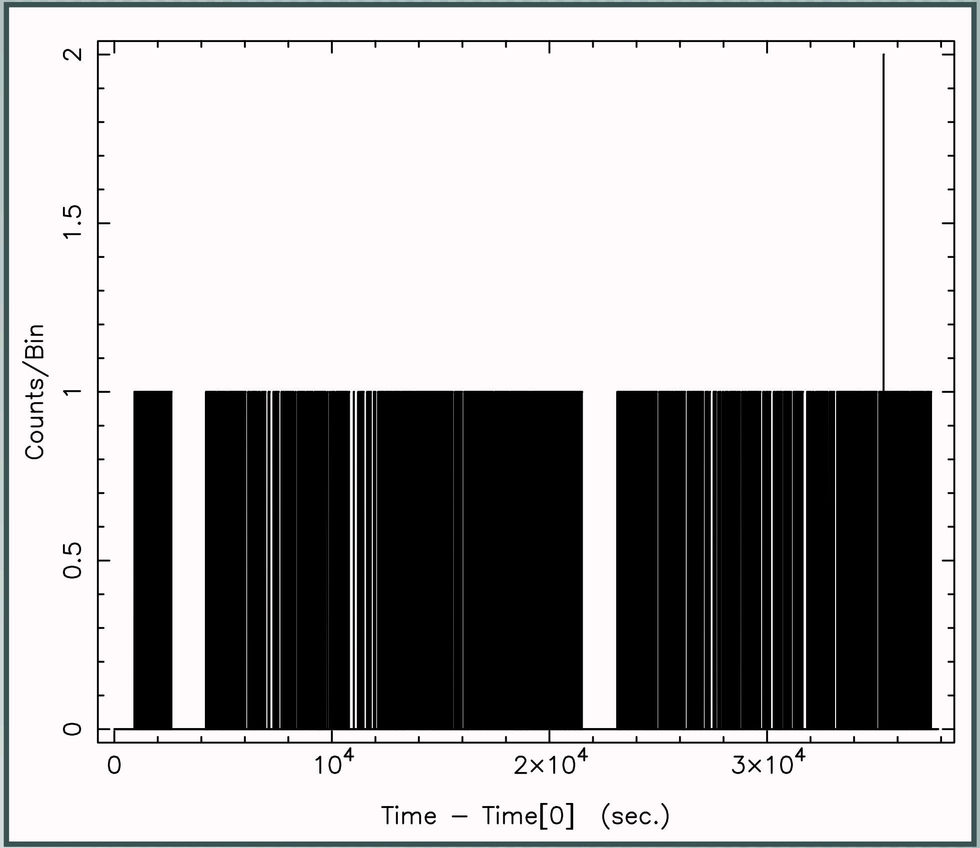


What are the Tools of the Trade?

- Spectra: XSPEC; Sherpa, ISIS - A Few Hardy Souls Run Their Own
- Timing: Xronos - Which Some People Use
- Most People “Roll Their Own”
  - Custom Fortran/C Code
  - IDL or MATLAB
  - Me: Converting over to S-lang run Under ISIS/Sherpa (http://space.mit.edu/CXC/analysis/SITAR - contributions welcome!)

Timing Starts with a Lightcurve

- Different Spacecraft can have different tools for creating Lightcurves
- ftools, dmtools, xselect
- Always choose integer multiple of "natural" time unit for binning
- Don't bin any more than you have to - save it for subsequent analysis
- Example: dmextract infile="4u2129_chandra.fits [EVENTS] [sky=region(source.reg)][bin time=::1.14104]" outfile=4u2129_ps.fits opt=ltc1



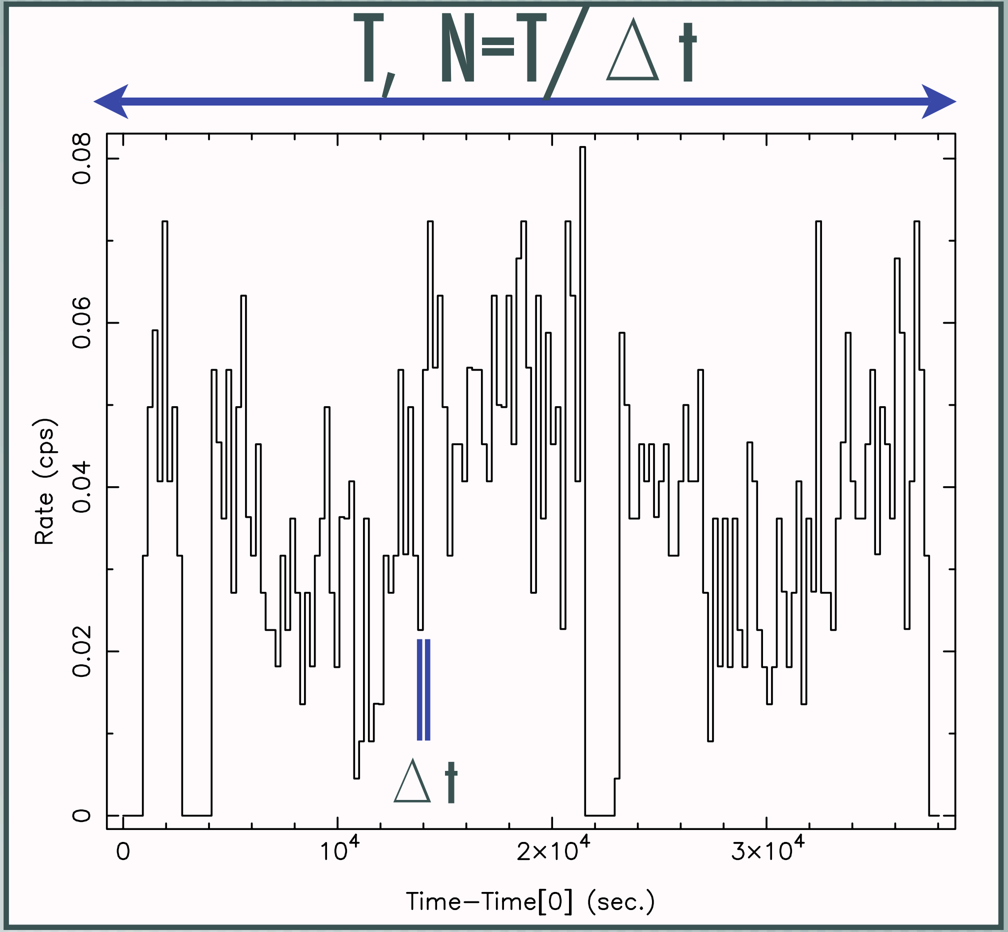
Length & Binning Determine Limits

- Lowest Frequency: $f_{long} = 1/T$
- Highest Frequency: Nyquist Frequency, $f_{Nyq} = 1/(2 \Delta t)$
- Basic Question, is the Variance:

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

Greater than Expected from Poisson Noise?

- σ = Root Mean Square Variability



Variability Test I: Excess Variance

Binned Lightcurve with Values: $X_i \pm \sigma_i$ and mean: μ

$$\sigma^2_{\mathrm{rms}} = \frac{1}{N\mu^2} \sum_{i=1}^{N} \left[(X_i - \mu)^2 - \sigma_i^2 \right]$$

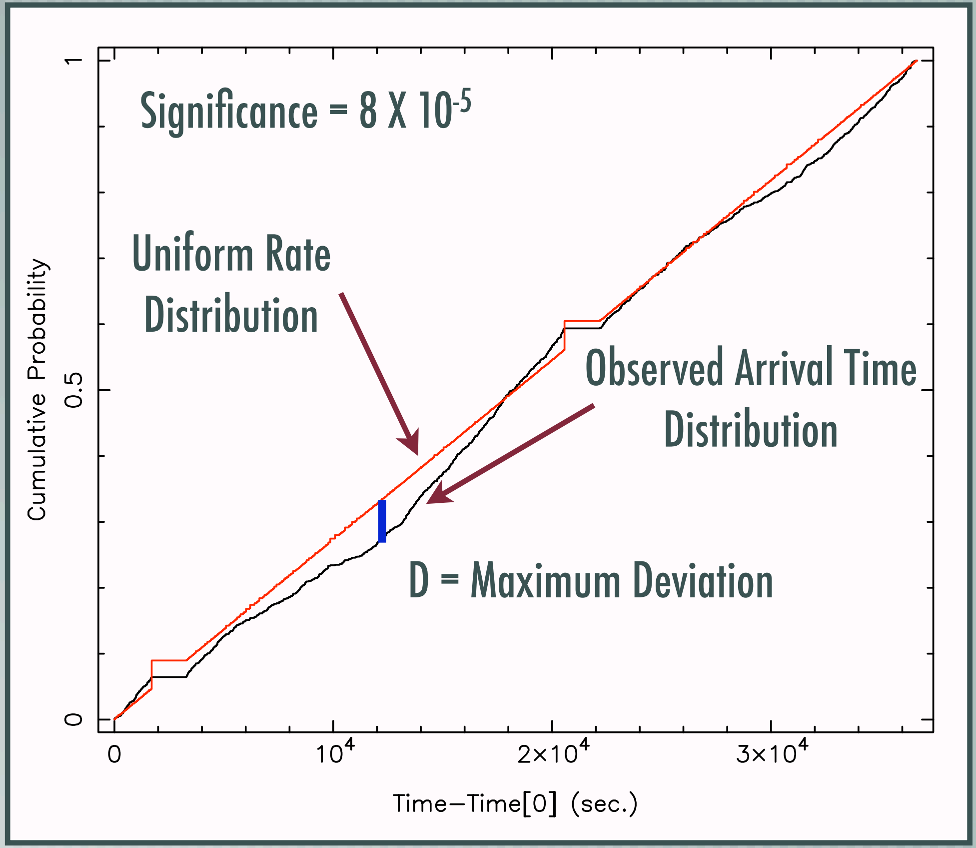
$$\Delta\sigma^2_{\mathrm{rms}} = s_D / (\mu^2 \sqrt{N})$$

$$s_D^2 = \frac{1}{N-1} \sum_{i}^{N} \left(\left[(X_i - \mu)^2 - \sigma_i^2 \right] - \sigma^2_{\mathrm{rms}} \mu^2 \right)^2$$

See Turner et al. 1999, ApJ, 524, p. 667 ; Nandra et al. 1997, ApJ, 476, p. 70

Test II: Kolmogorov-Smirnov

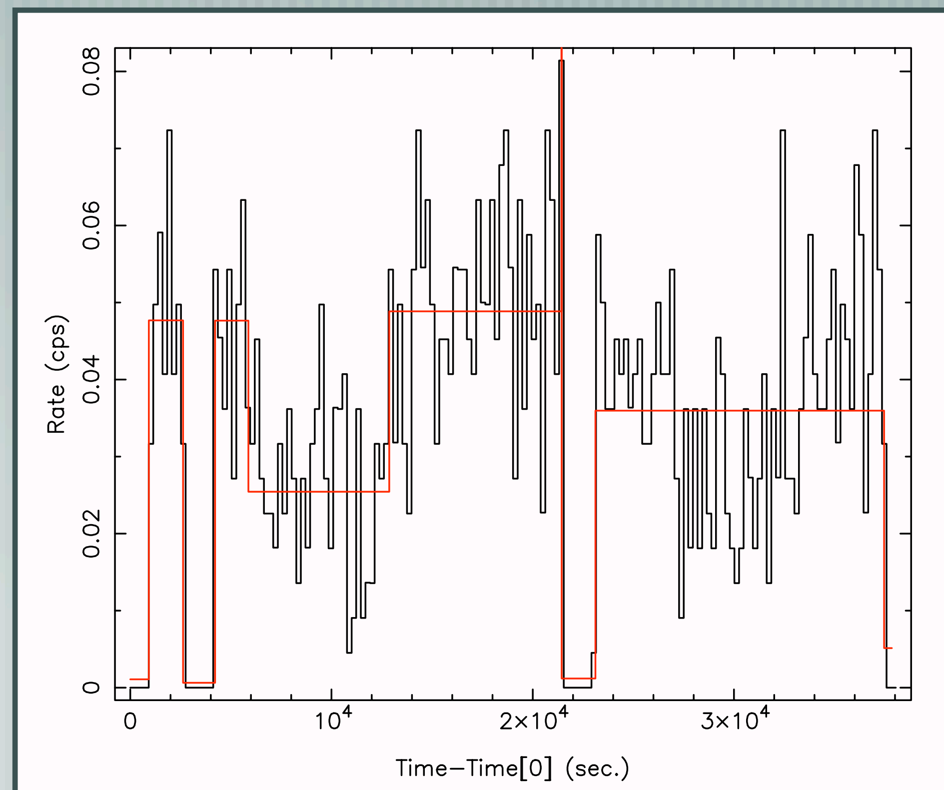
- Technique for determining whether two cumulative distributions are the same.
- Example: Is cumulative arrival time consistent with constant rate?
- Could have instead done distribution of times inbetween events.
- See Press et al., "Numerical Recipes", plus lots of other better statistics books
- Only answers whether there is variability - doesn't characterize it



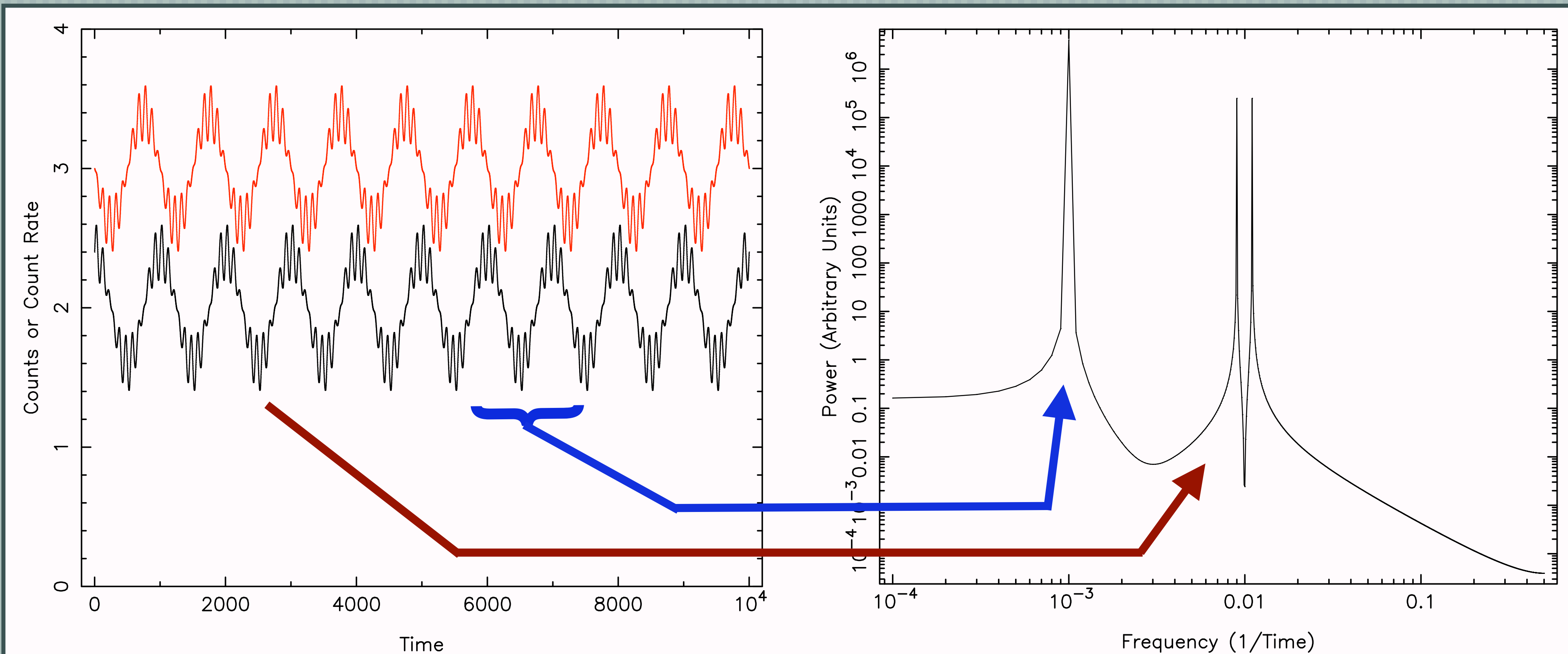
Ninja Topic: Bayes Stats

- Bayesian Methods Don't Require Binning (Case below: event times only!)
- Gregory & Loredo (1992, ApJ, 398, p. 146) - Determines Optimal Uniform Binning. (Eventually a Ciao Version)
- Bayesian Blocks (J. Scargle, in prep.) - Determines Optimal Non-uniform Binning. (S-lang Version on SITAR page)
- Drawbacks: No 'Frequentist' Significance Levels. Only 'Odds Ratios' or 'Penalty Factors'.

```
t = fits_read_col("4u2129_chandra.fits","time");
cell = sitar_make_data_cells(t,2,0.7,1.14104,min(t),max(t));
ans = sitar_global_optimum(cell,3.5,2);
```



Fourier Transform Methods



- The Workhorse of the Timing World
- How is Variability Power Distributed as a Function of Frequency?

Fast Fourier Transform (FFT)

$$X_j \equiv \sum_{k=0}^{N-1} x_k \exp(2\pi i j k/N) \quad , \quad j = [-N/2, \ldots, 0, \ldots, N/2]$$

$$P_j = 2|X_j|^2/(\mathrm{Rate}^2 \times \mathrm{T}_{\mathrm{total}})$$ (“One Sided” RMS Normalization)

$$P_j = 2|X_j|^2/(\mathrm{Rate} \times \mathrm{T}_{\mathrm{total}})$$ (“One Sided” Leahy Normalization)

- Lightcurve with: N bins, Comprised of Counts, x_i, becomes Power Spectrum, with N/2+1 independent Amplitudes, and N/2-1 independent Complex Phases (for Real Inputs)
- Good FFTs Usually Optimized for N = Power of 2 (RXTE Clock Runs in Powers of 2!)
- Know Your Normalization!!! Various FFT Routines Have Different Ones!
- Power Spectrum is the Squared Fourier Amplitude, Properly Normalized
- Power Spectrum is Throwing Out Information! Not Unique!

Ninja Topic: Convolution/Cross Correlation Theorem

$$h * g \ (t) \equiv \int_{-\infty}^{\infty} h(\tau) g(t+\tau) \ d\tau$$

$$G(f) \equiv \mathcal{F}\left[g(t)\right] \equiv \int_{-\infty}^{\infty} g(t) e^{-2\pi i f t} \ dt$$

$$\mathcal{F}\left[h * g\right] = F^*(f) G(f)$$

- Power Spectral Density (PSD, or Power Density Spectrum/PDS) is just the Fourier Transform of the Auto-correlation Function (i.e., h(t) = g(t)).
- Cross Power Spectral Density (CPD) is just the Fourier Transform of the Cross-correlation Function

FFT Normalizations

- Leahy: Poisson Noise Level = 2, Intrinsic Power Scales as Rate
- RMS: Intrinsic Power Independent of Rate, Noise Level = 2/Rate
- Integral of PSD is Measure of Root Mean Square Variability

$$A = \int P_{\rm rms} df = \sum_j P^j_{\rm rms} \Delta f \quad , \quad \Delta f = 1/T$$

$$\sqrt{A} = {\rm rms/mean} = \left(\frac{\langle x^2 \rangle - \langle x \rangle^2}{\langle x \rangle^2} \right)^{1/2}$$

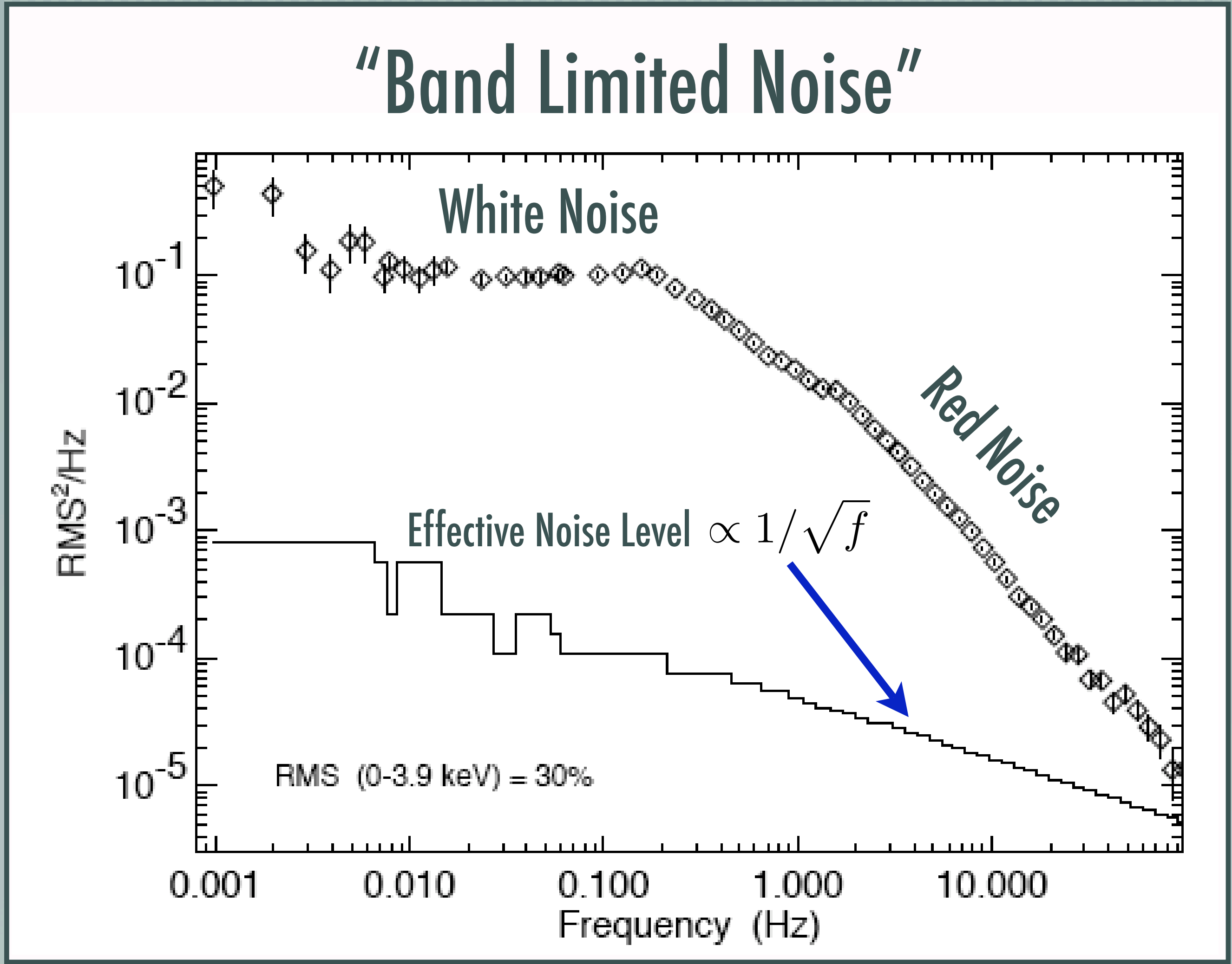
Pulsed Fraction (Coherent Oscillation): $f_p = \sqrt{\frac{2(P_{\rm Leahy} - 2)}{\rm Rate}}$

PSD Normalizations are Often Plotted as $({\rm RMS})^2/{\rm Hz}$

PSD Statistics

- Leahy Noise Level is 2 +/- 2 (Distributed as χ^2 with 2 DoF)
- Increasing Lightcurve Length Doesn't Help - Distributes Noise Among More Frequency Bins!
- "Statistically Stationary Processes" Have Power = P_i +/- P_i
- Reduce Noise by Averaging PSD from Individual Lightcurve Segments, as Well as Over (Usually Logarithmically Spaced) Adjacent Frequency Bins
- Errors Reduced by Factor of: $\sqrt{N_{\rm avg}}$

Example: Cyg X-1 (RXTE)

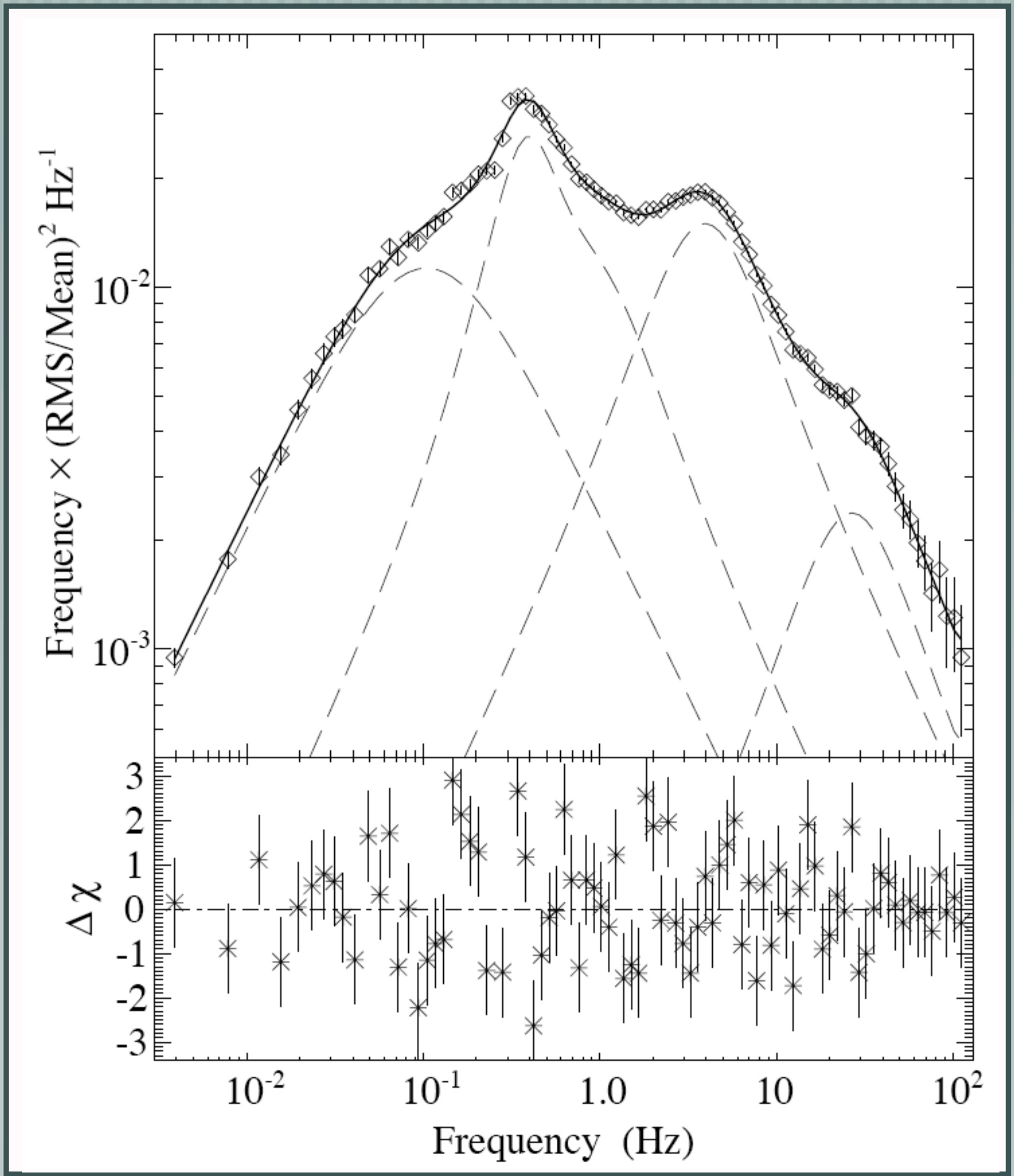


Nowak et al. 1999, ApJ, 510, p. 874

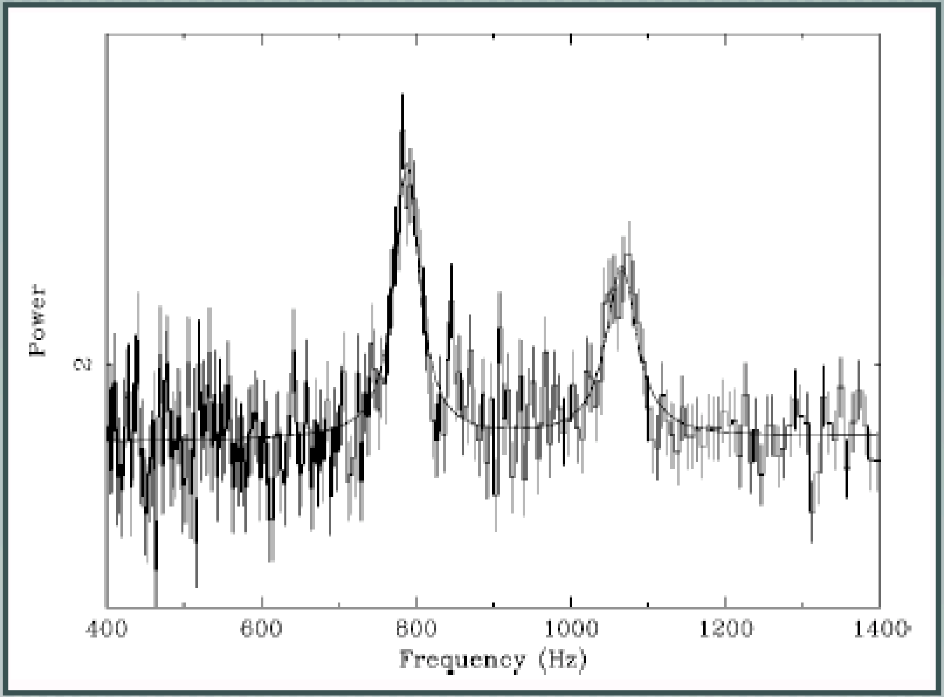
With: $P'_j = (P_j - P_{\rm noise}) \pm P_j/\sqrt{N_{\rm avg}}$ You Can Fit Models

Note: Total RMS = Incoherent Sum of Components, i.e., $\left(\sum_i \mathrm{RMS}_i^2\right)^{1/2}$

Advice: Fit Models that Average over Frequency Bin Widths



Quasi-Periodic Oscillations (QPO)



- Q-value (Coherence) = $f/\Delta f_{FWHM}$
- Width Can Come From: Finite Length of Data Segment, Finite Duration of Signal, Random Walk in Phase (e.g., Damped, Driven Oscillators), Random Walk in Frequency, ...

Ninja Topic: Deadtime

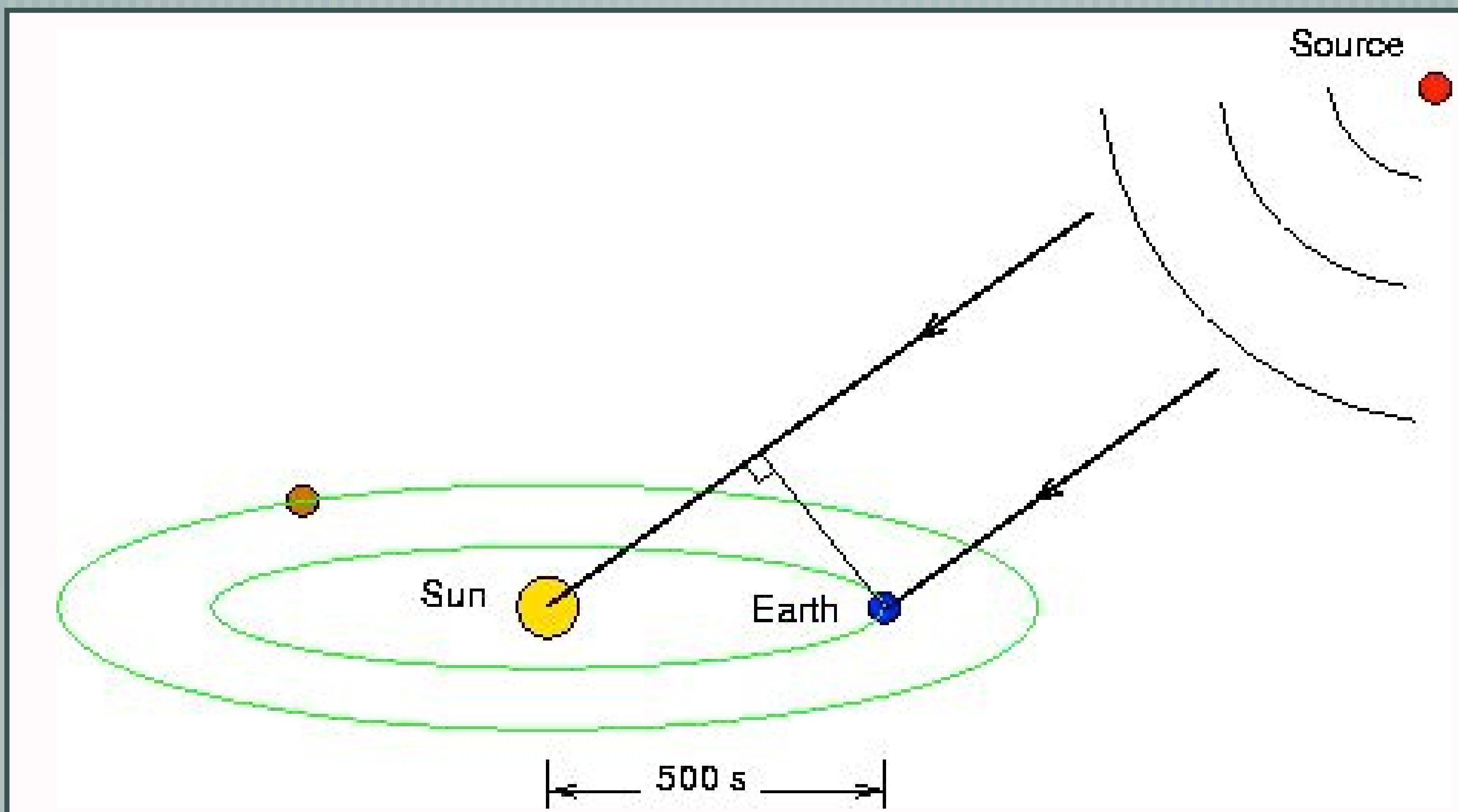
- Detector ‘Deadtime’ = When a Photon Event Prevents Subsequent Events from Being Detected (‘Paralyzable’ /‘Non-paralyzable’ is When an Event During the Deadtime Does/Does Not Increase the Length of the Deadtime), or ...
- When the Detector Does Not Take Data, e.g., during Readout (e.g., Chandra), or ...
- Deadtime Modifies the Power Spectrum of Poisson Noise from the Expected $P_{Leahy} = 2$ (Usually to Something < 2)
- See: Zhang et al. 1995, ApJ, 449, p. 930; Morgan et al. 1997, 482, p. 993; Nowak et al. 1999, ApJ, 510, p. 874

Proposal Estimates

- Detecting Broad Band Noise at the n_σ confidence level:
  - $\mathrm{RMS}^2_{\mathrm{limit}} \approx 2n_\sigma\sqrt{\Delta f}/\sqrt{\mathrm{Rate}^2 \times T_{\mathrm{total}}}$
- Detecting Coherent Pulsations:
  - $f_p^{\mathrm{limit}} = 4n_\sigma/(\mathrm{Rate} \times \mathrm{Time})$
- For Broad Band Timing, You Win More with Rate than Time
- Searches for Coherent Pulsations (e.g., Pulsars) are Best Done Unbinned

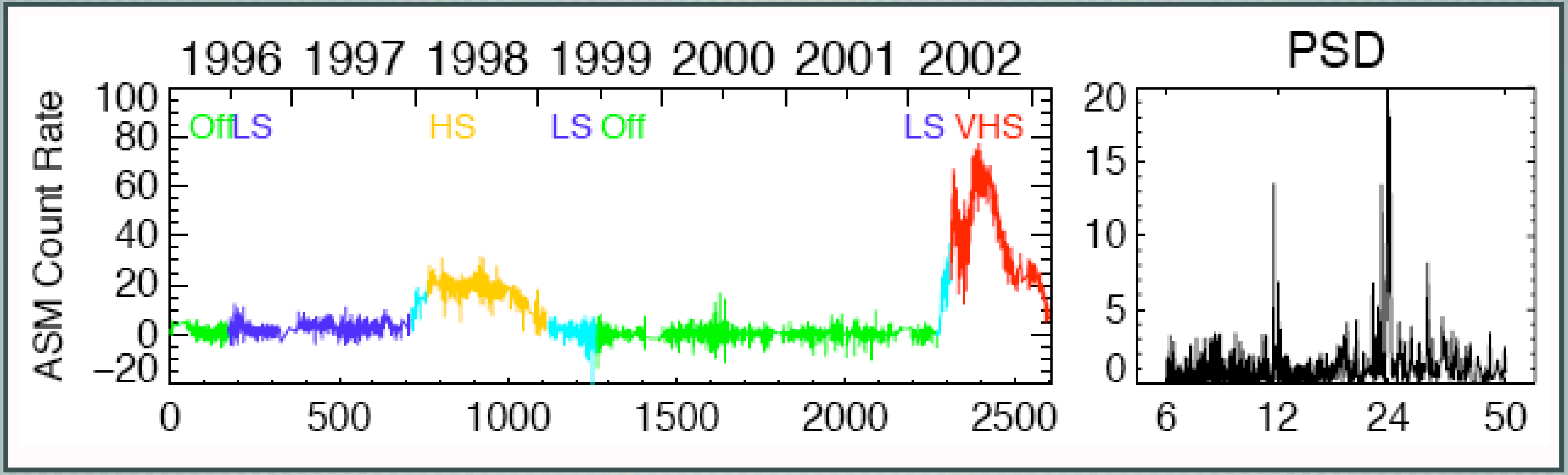
Coherent Pulsations:

- Barycentering the data (fxbary/axbary) important.
- Short Data Segments, to Search for (Binary) Orbital Variation



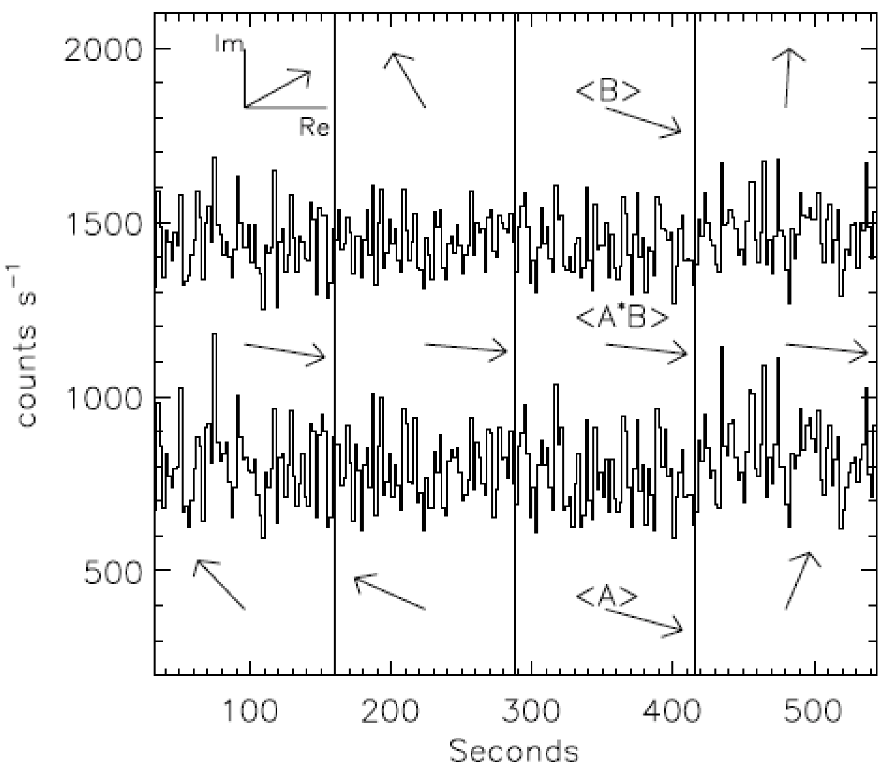
Ninja Topic: Aliasing!

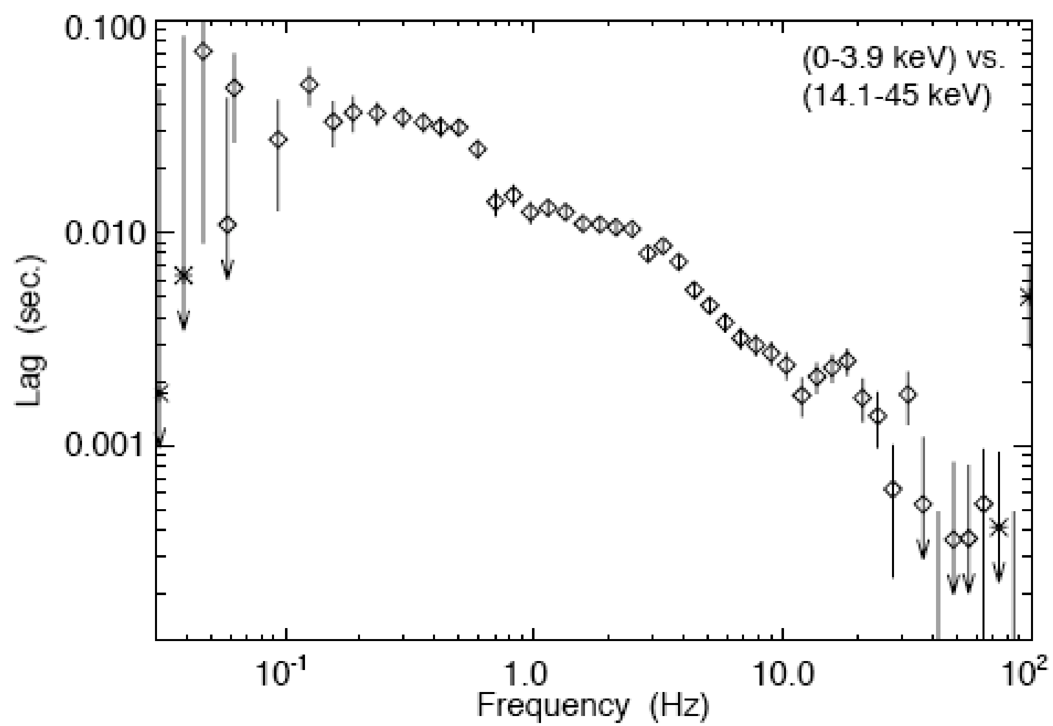
- Signal Can Appear at Sum and Difference Frequencies of Primary Signals
- This is True Whether the Signal is "Real" or "Fake" (e.g., Sampling Periods)
- Beware Characteristic Times! Spacecraft orbits, dither time scale, 1 year, ...
- Example: RXTE-All Sky Monitor - Many sources show periods at 24 hours +/- a small bit. This is the beating of a large power 1/Many Year Secular Change with a 24 hour sample Period (e.g., from AGN monitoring).



Ninja Topic: Phase Info

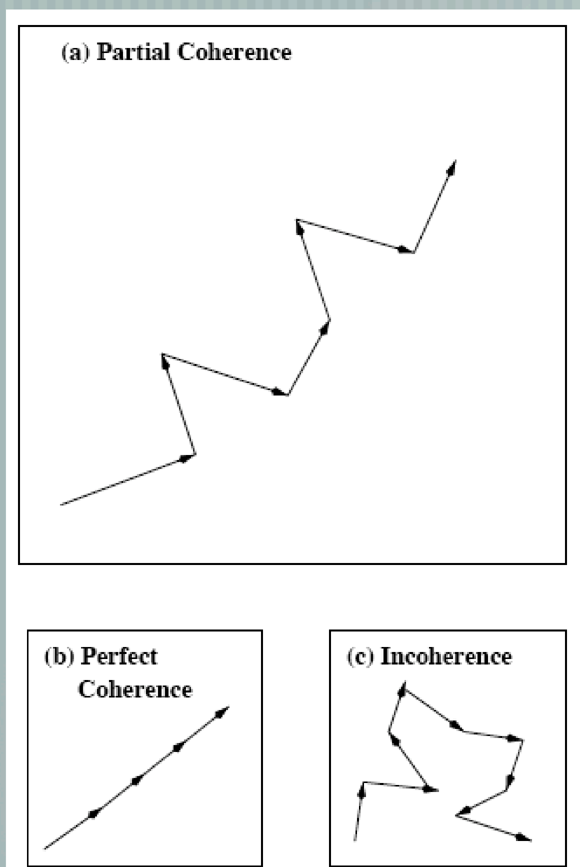
- There are Statistics That Also Deal with Fourier Phase - Cross Correlations!
- Gives the Frequency Dependent Time-lag between Hard and Soft Components
- See: Vaughan & Nowak 1997, ApJ, 474, L43; Nowak et al. 1999, ApJ, 510, p. 874





Ninja Topic: Cross Correlations

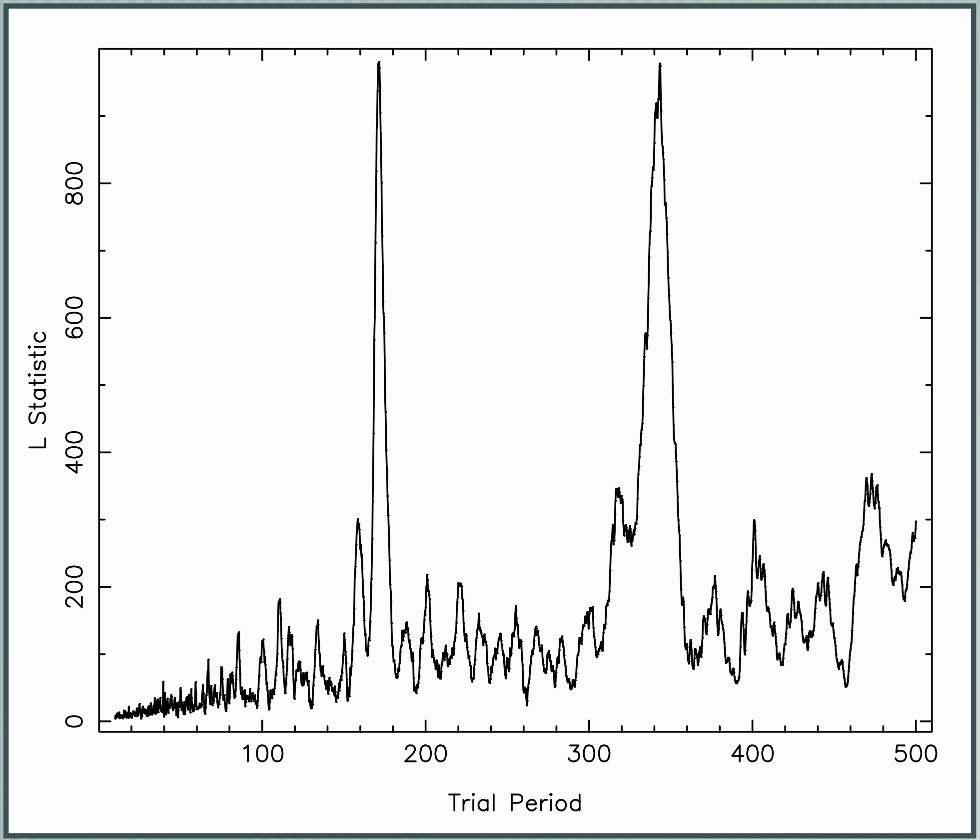
$$\langle CPD \rangle = \frac{\langle H^* G \rangle}{\left(\langle |H|^2 \rangle \langle |G|^2 \rangle\right)^{1/2}} \quad , \quad \gamma^2(f) \equiv \frac{|\langle H^* G \rangle|^2}{\langle |H|^2 \rangle \langle |G|^2 \rangle}$$



- Complex Phase is Called the "Phase Lag", Divided by 2πf is Called the "Time Lag"
- Keep track of signs! Depending upon Algorithm, and Whether You Use Forward or Backward Transform, that Can Alter the Sign. (See Nowak et al. for Associating this with Lag/Lead.)
- Υ^2(f) is the "Coherence Function" (Distinct from Coherence, Q!). Measures Degree of Linear Correlation.

Epoch Folding & Period Searches

- Good for Non-sinusoidal Variations
- Good for When there are Data Gaps or Complicated Window Functions
- Not Good for Aperiodic Variability




```
event = sitar_readasm("xa_x1820-303_d1",,,1.2);
fld = sitar_epfold_rate(event.time,event.rate,10,500,20,2000);
xlabel("Trial Period"); ylabel("L Statistic");
plot(fld.prd,fld.lstat);
```

Xronos has epoch folding, various IDL routines can be found on the web.

Read the literature on significance levels!

Reiterating Words of Advice:

- Bin the Lightcurve on Integer Multiples of "Natural" Time Scales
- Do FFTs with Evenly Spaced Bins (Lomb-Scargle for Unevenly Spaced Bins), and Avoid Data Gaps (see literature if dealing with Gaps)
- Beware of Signals that Appear on Characteristic Time Scales (of Spacecraft, Earth, etc.)
- Large Literature with Many Techniques for Those with Strong Kung Fu

References for Further Reading

- van der Klis, M. 1989, “Fourier Techinques in X-ray Timing”, in Timing Neutron Stars, NATO ASI 282, Ögelman & van den Heuvel eds., Kluwer
- Press et al., “Numerical Recipes” (Discussions Only! Better Code Exists on the Web!)
- Leahy et al. 1983, ApJ, 266, p. 160 (FFT & PSD Statistics)
- Leahy et al. 1983, ApJ, 272, p. 256 (Epoch Folding)
- Davies 1990, MNRAS, 244, p. 93 (Epoch Folding Statistics)
- Vaughan et al. 1994, ApJ, 435, p. 362 (Noise Statistics)